

Real-Time Implied Volatility Surfaces: A Practitioner's View of the Black Arts

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Introduction

- The listed options market has grown dramatically over the last two decades.
 - In US alone: circa 1,600,000 options on 5,600 underliers (OPRA, Oct 2023)
 - SPX has about 20,000 options (calls and puts) and about 60 expiries these days!
 - Daily ODTes, retail trading, meme stocks and many other new market “features”...
- Prop shops and hedge funds are much more important players now. OTC flow and exotics markets can’t ignore the listed (vanilla) market (but still try?).
 - Prop shops trade autocallables & hedge funds have HFT teams !?
- Vanilla valuation is complicated due to divs, borrow costs, rate term-structure, events, settlement details, Berm/Amer, vol-time, and “funky” vol curve shapes.
 - American “vanillas” are really path-dependent exotics — getting EEP right is hard!

Introduction (cont'd)

- These problems are hard at any time scale — but especially if all possible corner cases have to be handled in real time!
- The two main threads of this talk are:
 - a. The listed options market has become very “sophisticated and opinionated”. It contains a lot of useful information.
 - b. The many modeling and algorithmic details one has to get right to create and maintain a large-scale, real-time valuation & risk (+ margin, PnL decomp, etc) infrastructure.
- Can only give a flavor of the myriad of details one has to deal with... but it's a 2-day conference and I have a lot of (extra) slides we can discuss.

Implied Vol Surfaces

- **Implied volatility surfaces & borrow/forward curves** are the standard approach to summarizing the vanilla options market in an intuitive and compact manner.
- They provide the fundamental building block for the trading of vanillas (listed and OTC), as well as flow derivatives and exotics.
- There are many quant problems facing options and derivatives trading desks, and the problem of **constructing sensible, arbitrage-free volatility surfaces from options market prices** (bids and asks) is one of the hardest.
- This issue already exist for European-style options (SPX, VIX, SX5E, DAX, etc).

Implied Vols and (American) Pricing

- For European options (without divs) only integrated rates and variances matter.
 - Cash dividend modeling is relatively minor issue for Euro options (unless stochastic divs...).
- But **American** options are really path-dependent exotics and a lot of extra complications arise (esp. for ETFs, stocks, esp. with dividends):
 - Need to choose proper cash dividend and borrow cost modeling. Then:
 - Even in BS: Rate term-structure, proper choice of “vol time” (including events), and details like settlement, Bermudan vs American all significantly affect early exercise premia (EEP).
 - Beyond BS: Local vol? Stochastic LV? (Look at volga, vanna...)
 - Approxs/hacks to adjust ITM relative to OTM vol to still price call & put of same strike in BSM.
- There are subtleties in “De-Americanization” (see above and at the end...), but if in doubt think of “implied vol surfaces” as summarizing European options prices in a convenient and intuitive manner (whether they are listed/traded or not).

Vol Curve/Surface Parametrizations

- There are advantages to having a good vol curve parametrization (by strike).
- For current purposes all we have to know is:
 - There are various curves in the public domain, e.g. **S* curves**:
 - i. SVI / S5 (5 params per term)
 - ii. SSVI / S3 (3 params per term)
 - iii. SABR (3 params per term; name is overloaded: model and curve...)
 - The S* curves do not have much shape flexibility, e.g. they do not allow W-shapes as required around events.
 - Hence there are many proprietary curves out there... Vola has **C* curves**.
 - First 3 params: **vol0** (=ATF vol) and **s2**, **c2** (dimensionless slope & curvature...).

Other Inputs for Pricing: Forwards, Rates, Divs

- To price European options of any expiry and payoff we need:
 - A discount rate curve: Term rates $r(T)$
 - A forward curve: $F(T)$ (also divs in some models of pricing Euros...)
 - An implied vol surface (IVS aka VS).
- Convenient to think of the **forward curve** in terms of **rates** and **divs** (0 if Fut, FX)
 - In the **American** case, the **rates are primary**, i.e. needed for proper EEP calculation.
 - The forward grows with the “drift” between divs, and jumps down by the div amount at ex-div dates. “drift” = fundingRate – borrowRate = $b - q$:

$$F_T = f_p(T) \left(S_0 - \sum_{i:t_i \leq T} \frac{d_i}{f_p(t_i)} \right) \quad f_p(t) := \exp \left(\int_0^t (b(t') - q(t')) dt' \right)$$

NB: Sometimes $b=r$ is used, since the difference can be absorbed into the (implied) borrow rate q .

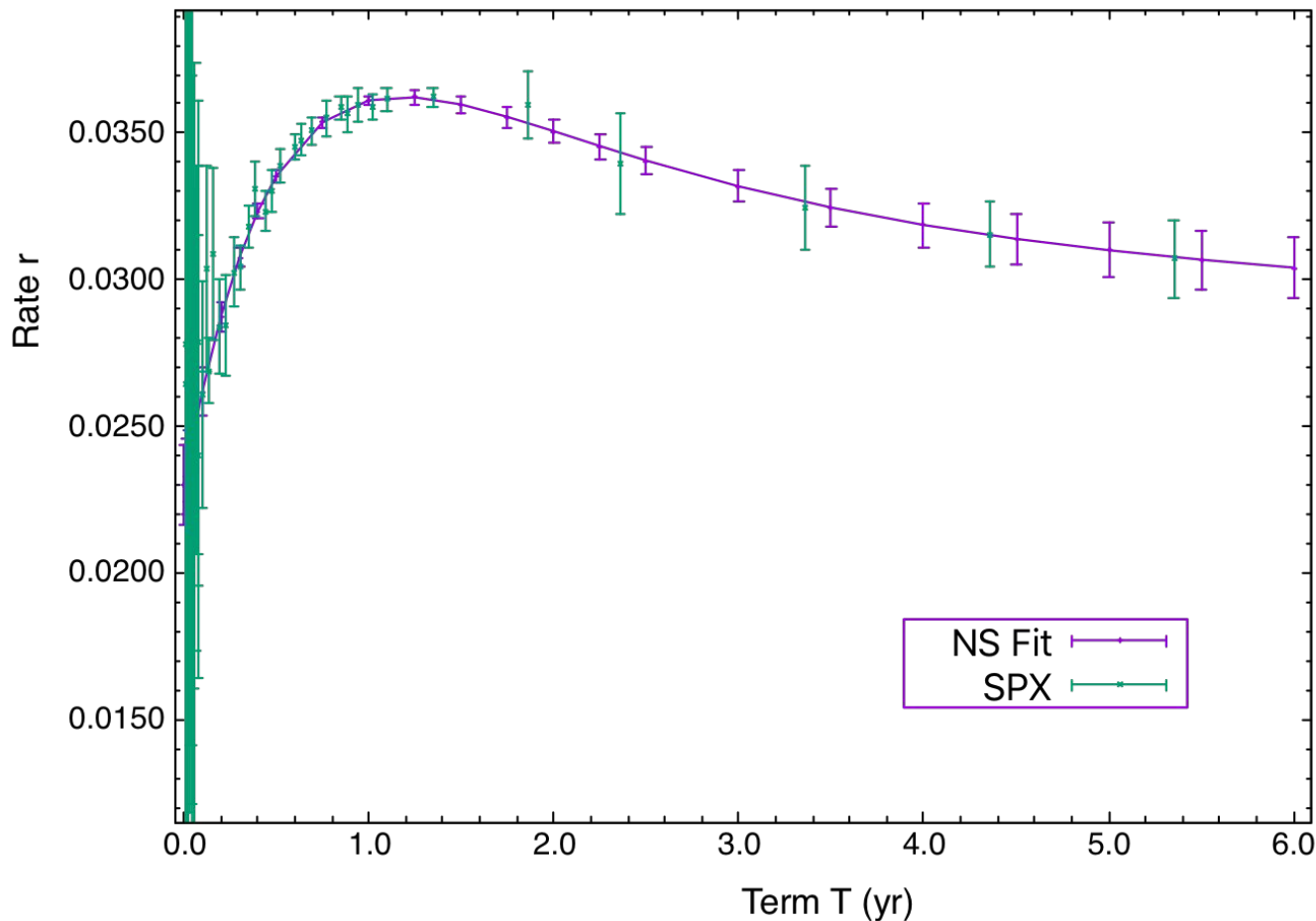
“SPIBOR” — SPX Implied Discount Rates

- What discount curve should one use for options pricing and trading?
 - Depends... but for implying borrows, vols, etc, use **market consensus**.
- European put-call-parity (PCP) for a given term:
 - $$C - P = DF \cdot F - DF \cdot K$$
- To imply the **discount factor** for a given term T, $DF = DF(T)$, we need a robust linear regression across many strikes K.
- For further robustness, can smooth rates across T via a term-structure fit.

SPIBOR — Even the Fed cares now!

- Fed (-associated) economists have written a number of papers about SPIBOR in the last few years.
- Why does the Fed care?
 - The Fed needs to know what's going on...
 - **Treasury, SOFR, etc rates are NOT risk-free rates!**
 - They can be lower than risk-free ("convenience yield"), or higher ("default risk").
 - Usually they are a bit lower, by 20 – 40 bps (almost flat).
 - SPX options market makers should be using close to risk-free rates ("box rates") due to margin requirements at exchange and OCC level.

Options-Implied Discount Rates 20220811-130000, chi2Red=0.170



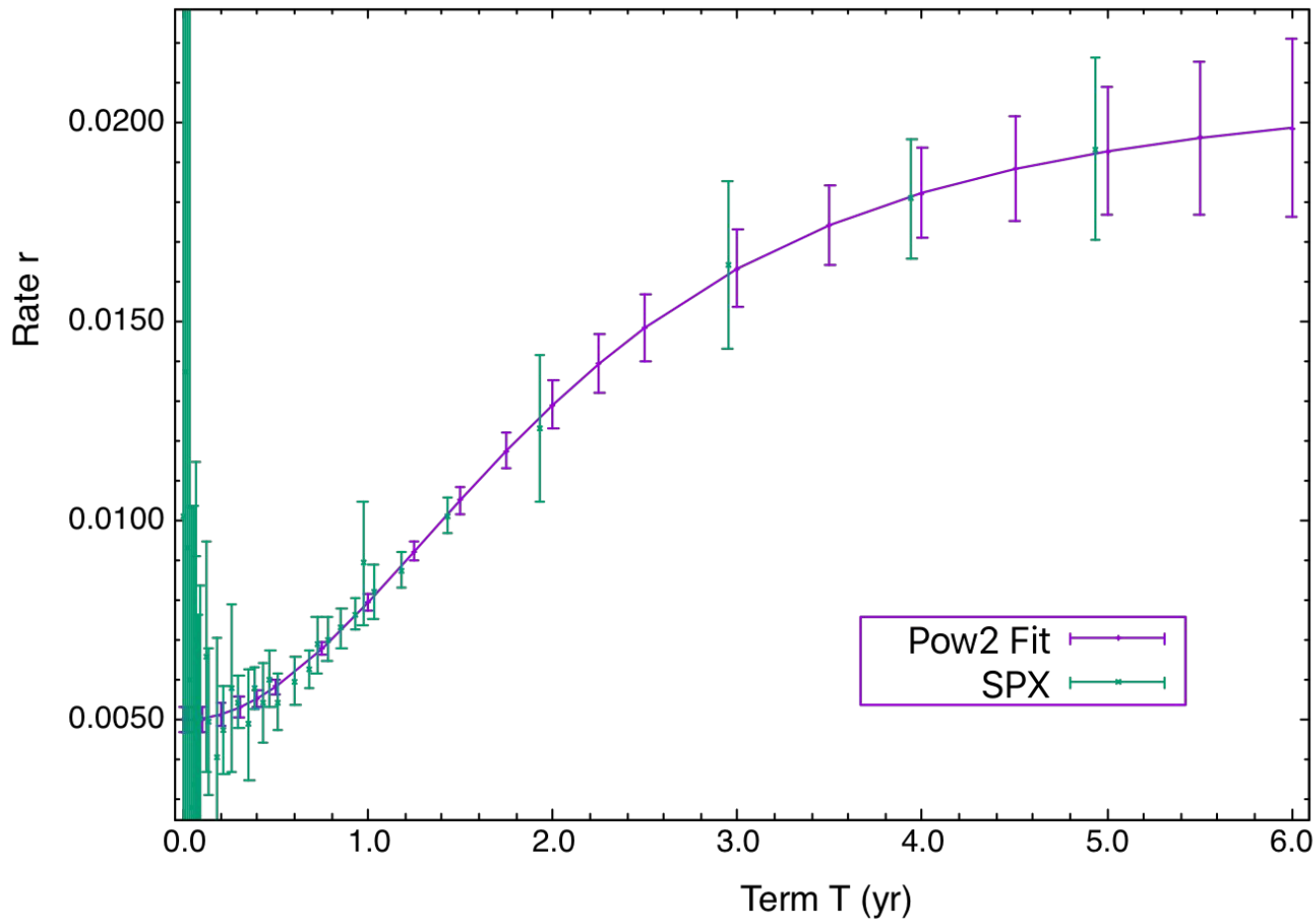
What discount rates should I use?

SPIBOR

Just one snapshot!

Nelson-Siegel TS fit

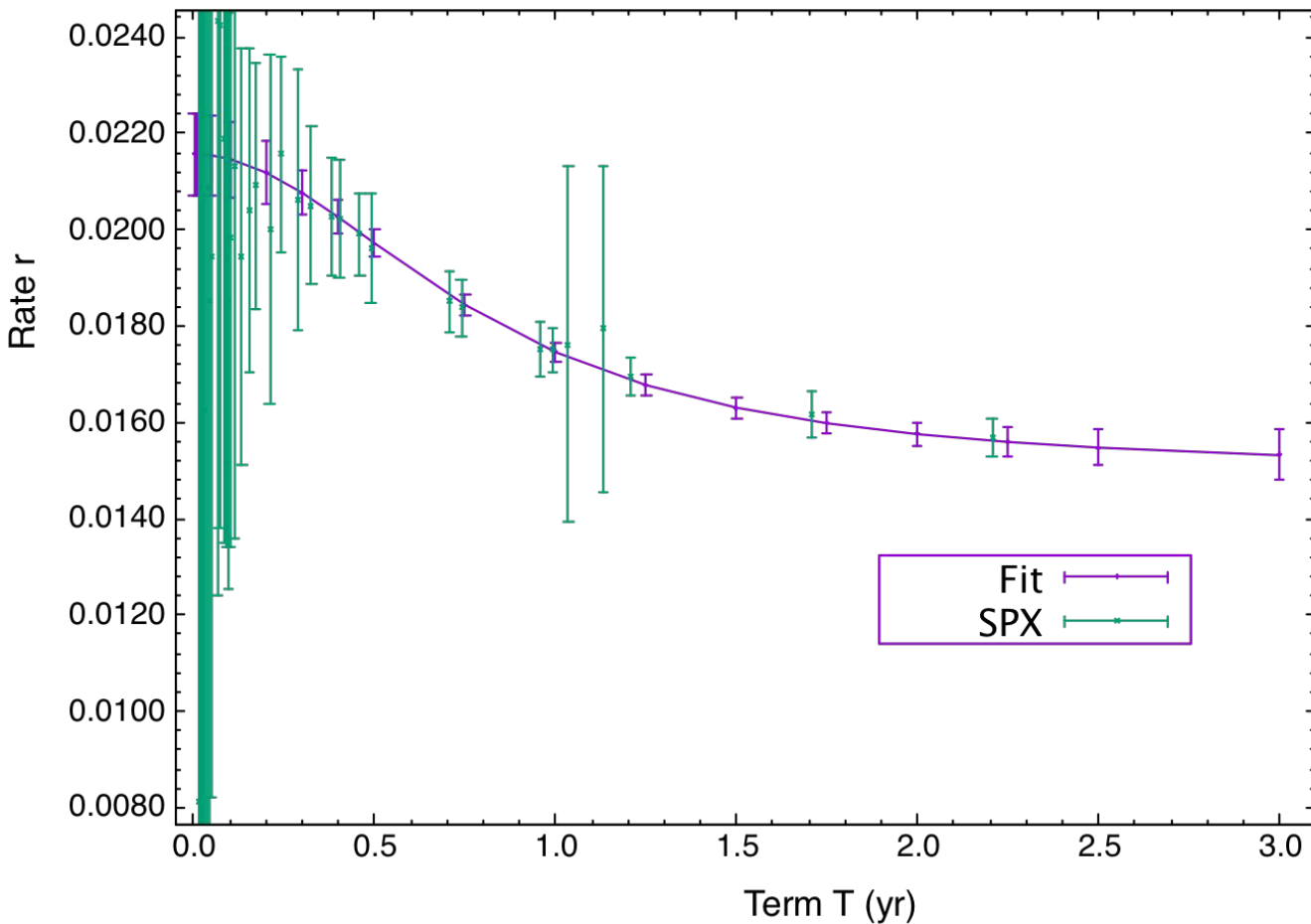
Options-Implied Discount Rates 20220110-150000, chi2Red=0.119



What discount rates should I use?

SPIBOR

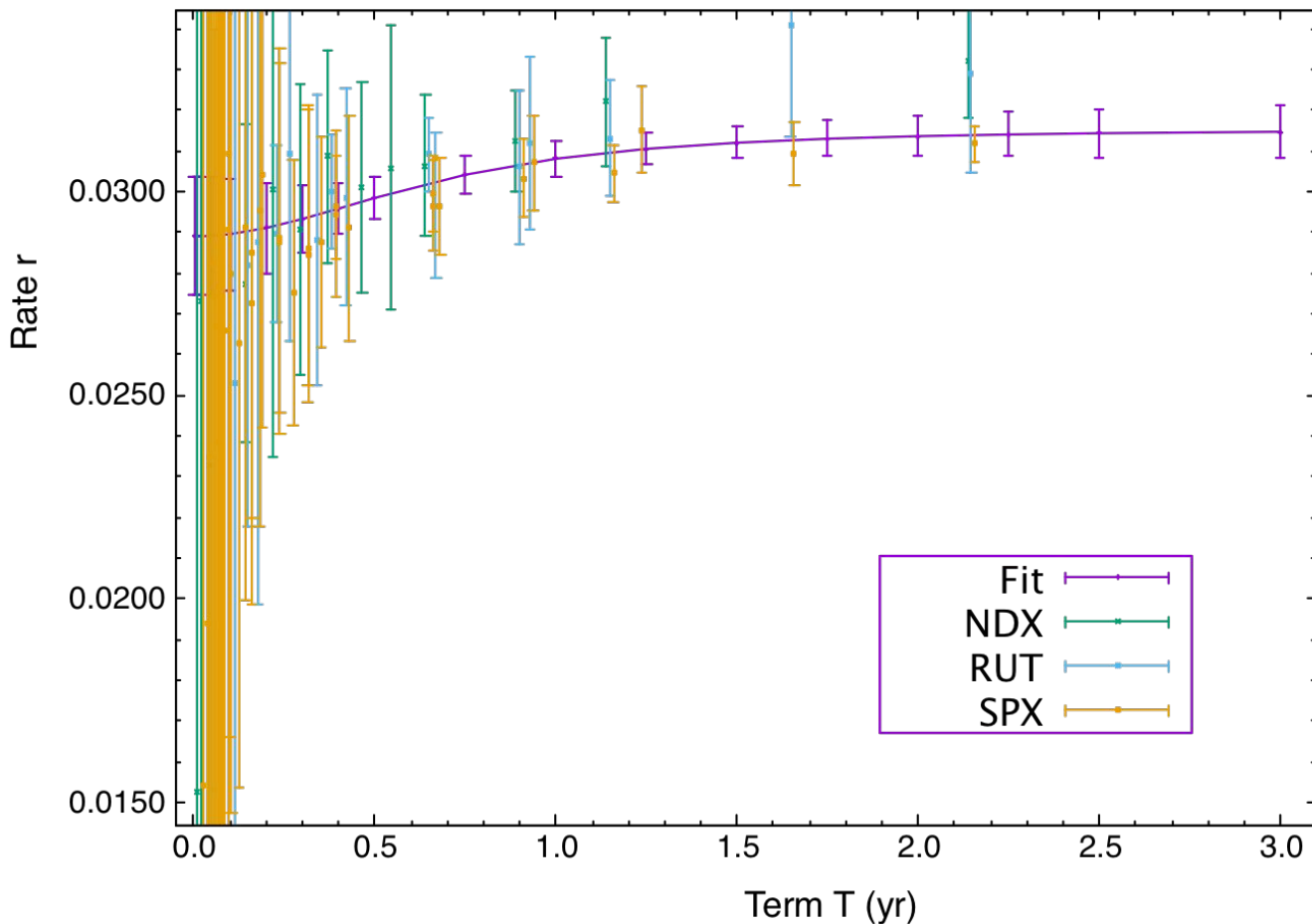
Options-Implied Discount Rates 20191004, chi2Red=0.077



What discount rates should I use?

SPIBOR

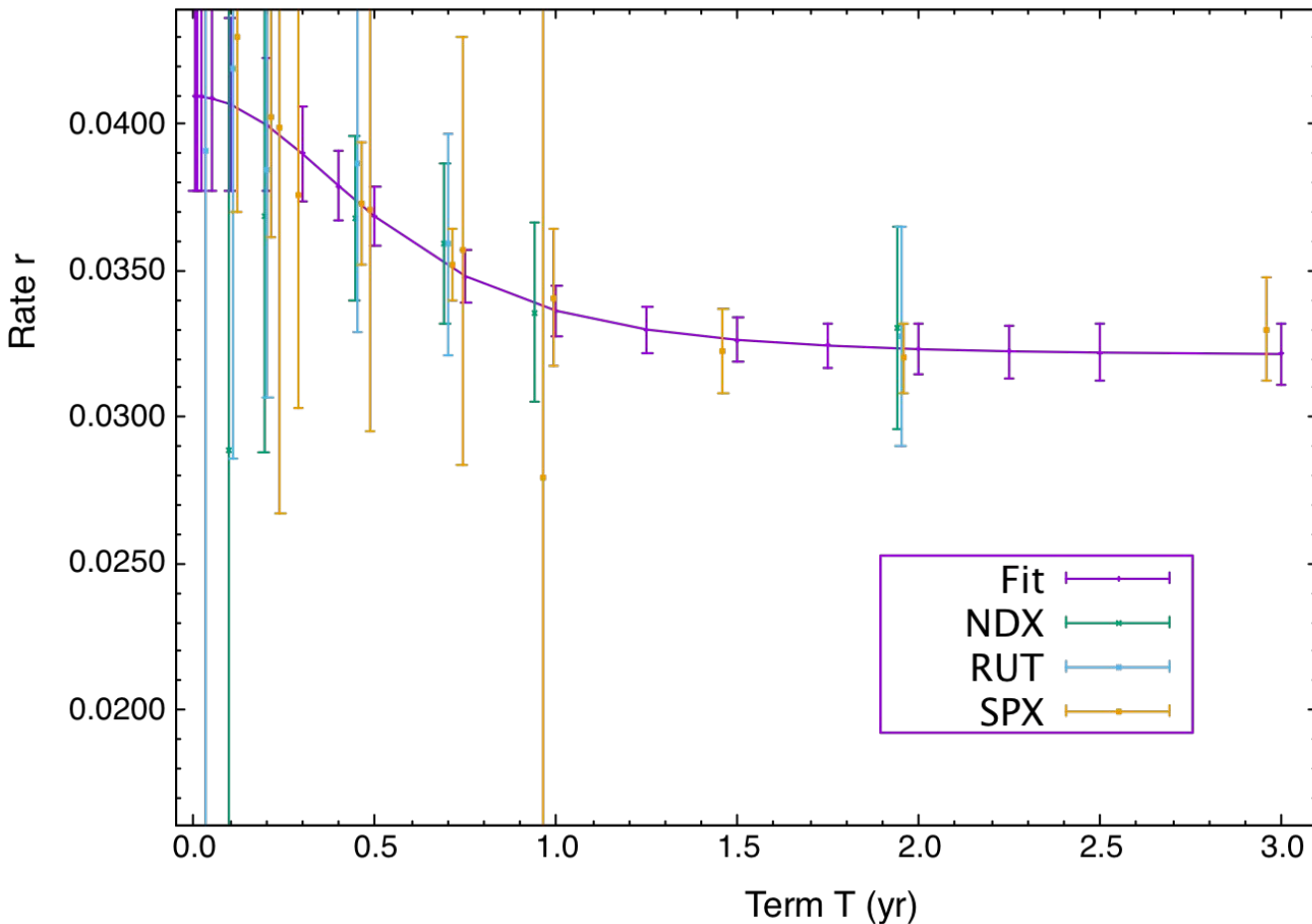
Options-Implied Discount Rates 20181030, chi2Red=0.302



What discount rates
should I use?

Maybe they are underlier/
sector dependent?

Options-Implied Discount Rates 20080111, $\chi^2_{\text{Red}}=0.140$



What discount rates
should I use in **2008** ??

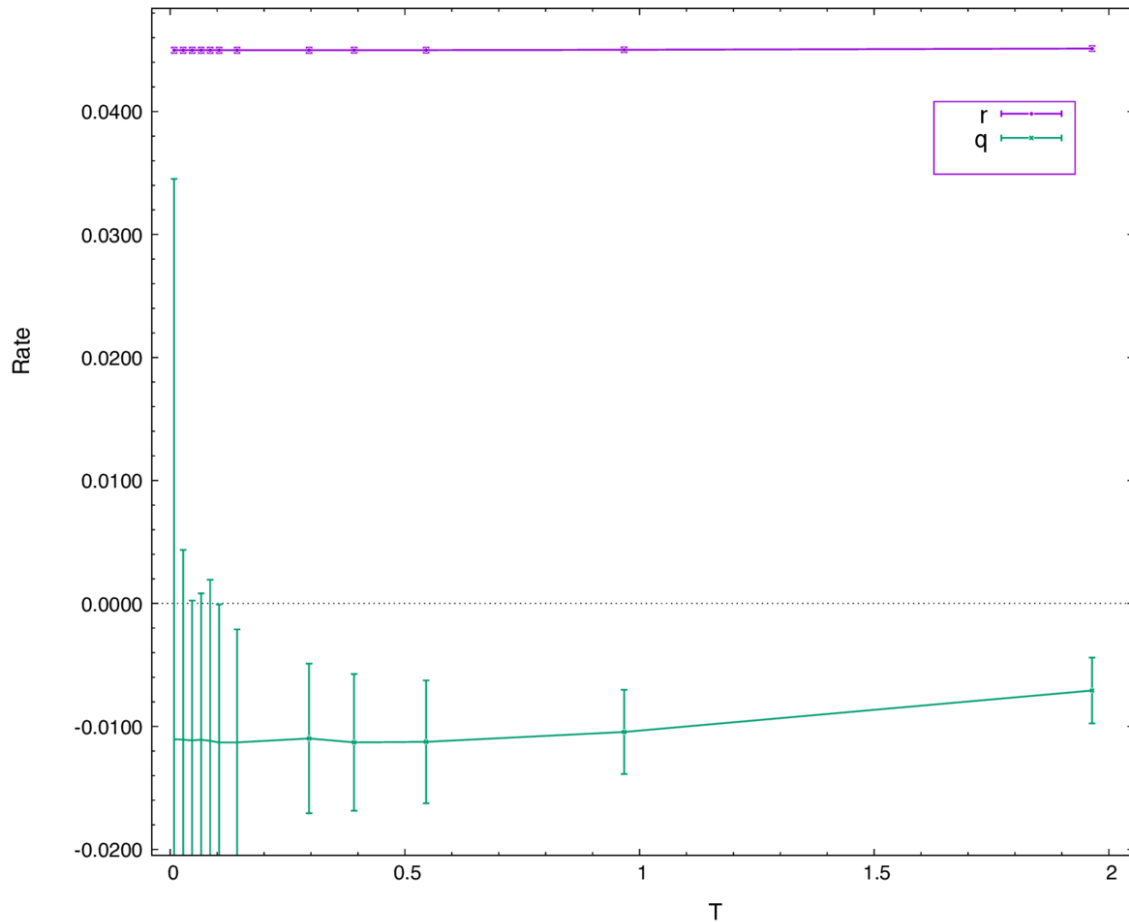
SPIBOR

Example: IBIT options

- IBIT = iShares Bitcoin ETF, launched 2024-02-15.
- One of the most successful ETF launches in decades: ~\$80bn AUM (Sep 2025)
- Options started trading 2024-11-19: Very quickly became very liquid.
- We will show:
 - Implied borrow costs $q(T)$.
 - Vol fits: SVI vs C9W, with metrics like **chi2Reds**, **avE5** (avgErrors5)
 - Theoretical prices (“theos”) vs market prices
 - No arbitrage: Densities, total variances

How to imply the borrow cost $q(T)$?

- For European-style options we could use PCP to imply the forward for each expiry, and then $q(T)$ from the forwards.
- For American-style options PCP does not hold. Call and/or put prices could have an early-exercise premium (EEP), even around ATM.
- Time-honored tradition is to use “**American PCP**” as:
 - For strikes K around ATM demand: $\text{volPut}(K) = \text{volCall}(K)$
- This is not really true! One can check this in any real model, like LV, SLV.
- It holds “well enough” if EEP are small-ish compared to spreads for some strikes around ATM — OR if this is what “the market” does!
- For now, imply the borrow such that American PCP holds.
 - If pricing with rate term-structure, **bootstrapping** from small to large T is needed.



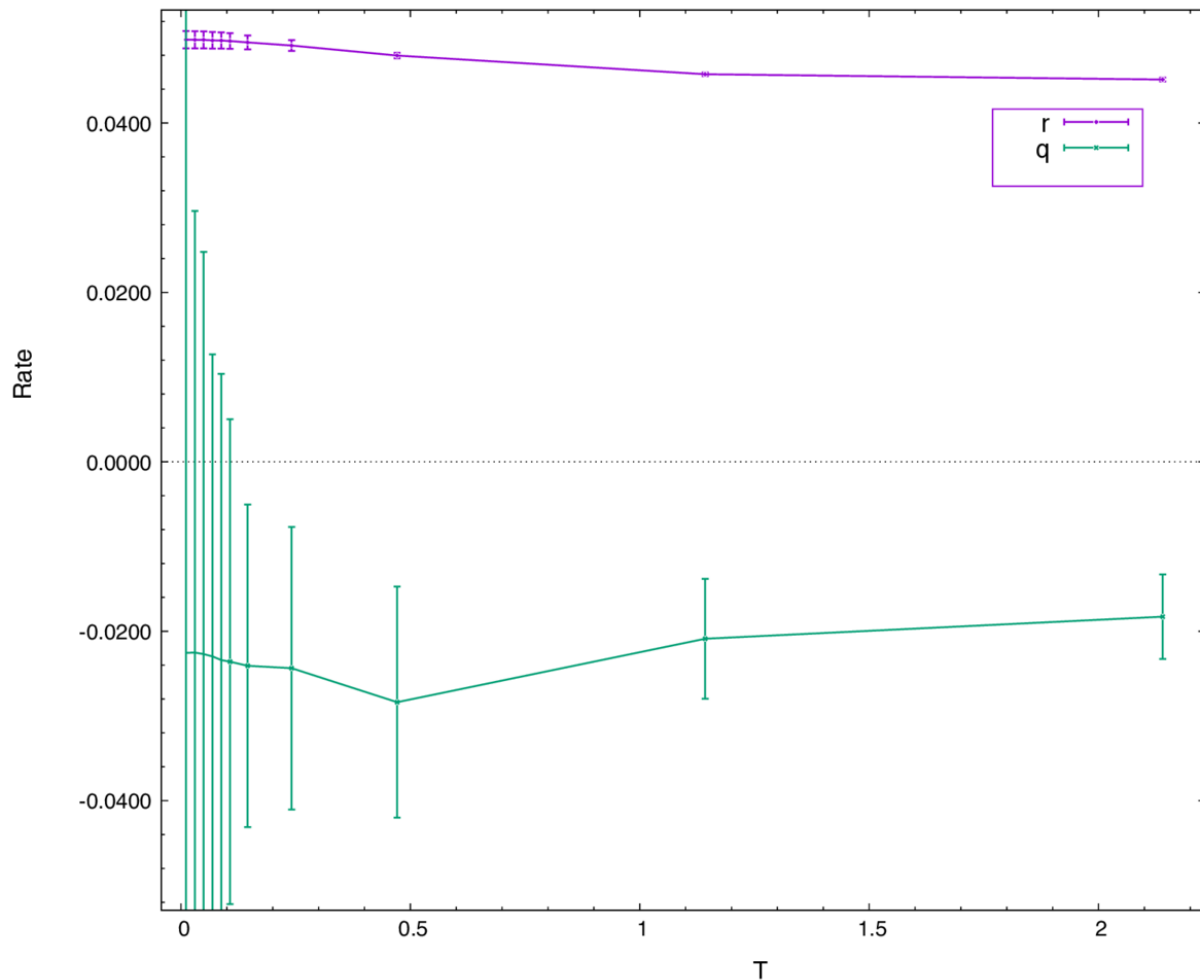
IBIT 2025-01-28

Implied borrow cost by term $q(T)$

The actual borrow cost is +1.2%

Massive demand for upside leverage
leads to much smaller implied $q(T)$!

We also implied the spot here.



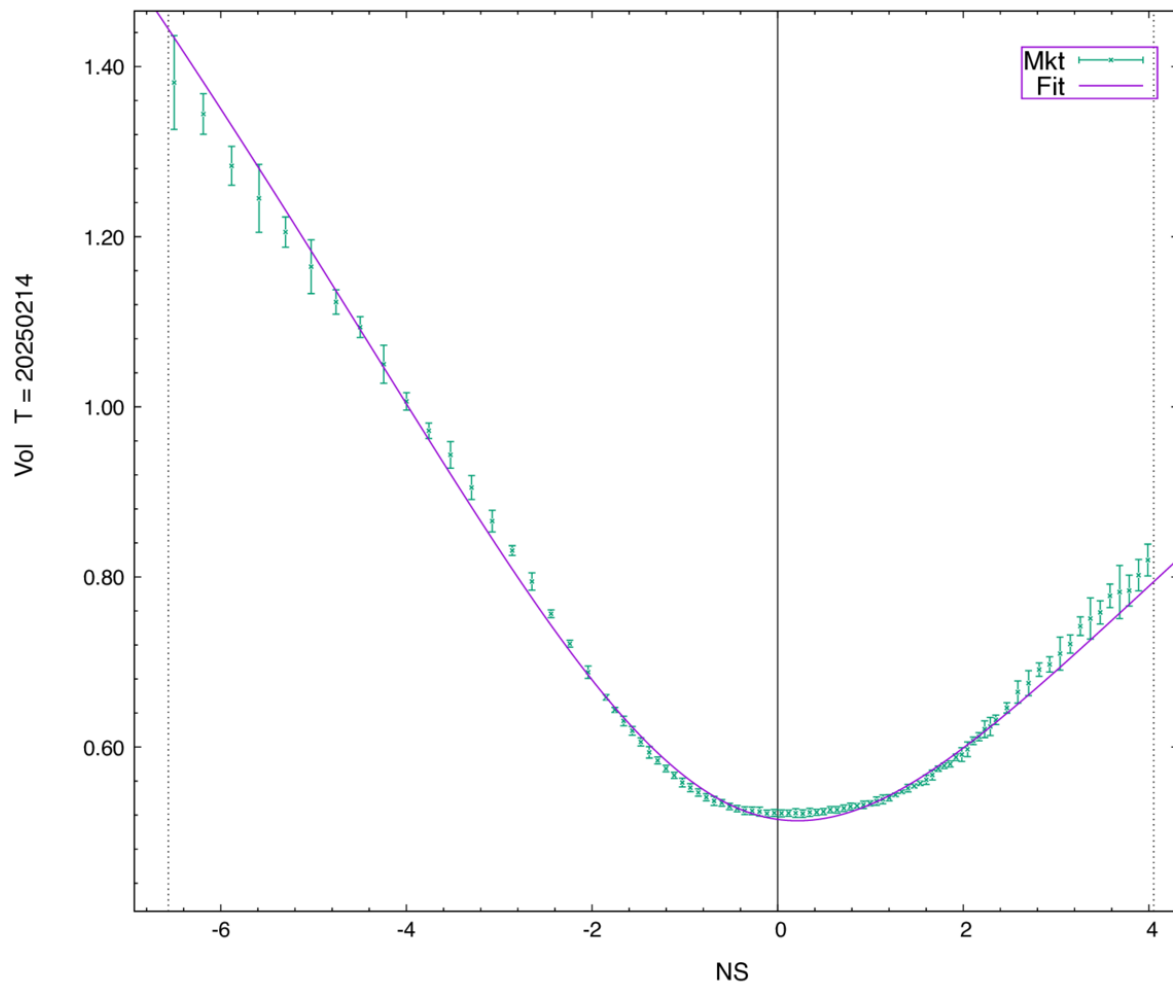
IBIT 2024-11-25

A few days after options trading started

Implied borrow cost by term $q(T)$

Massive demand for (upside leverage)
leads to much smaller implied $q(T)$!

We also implied the spot here.

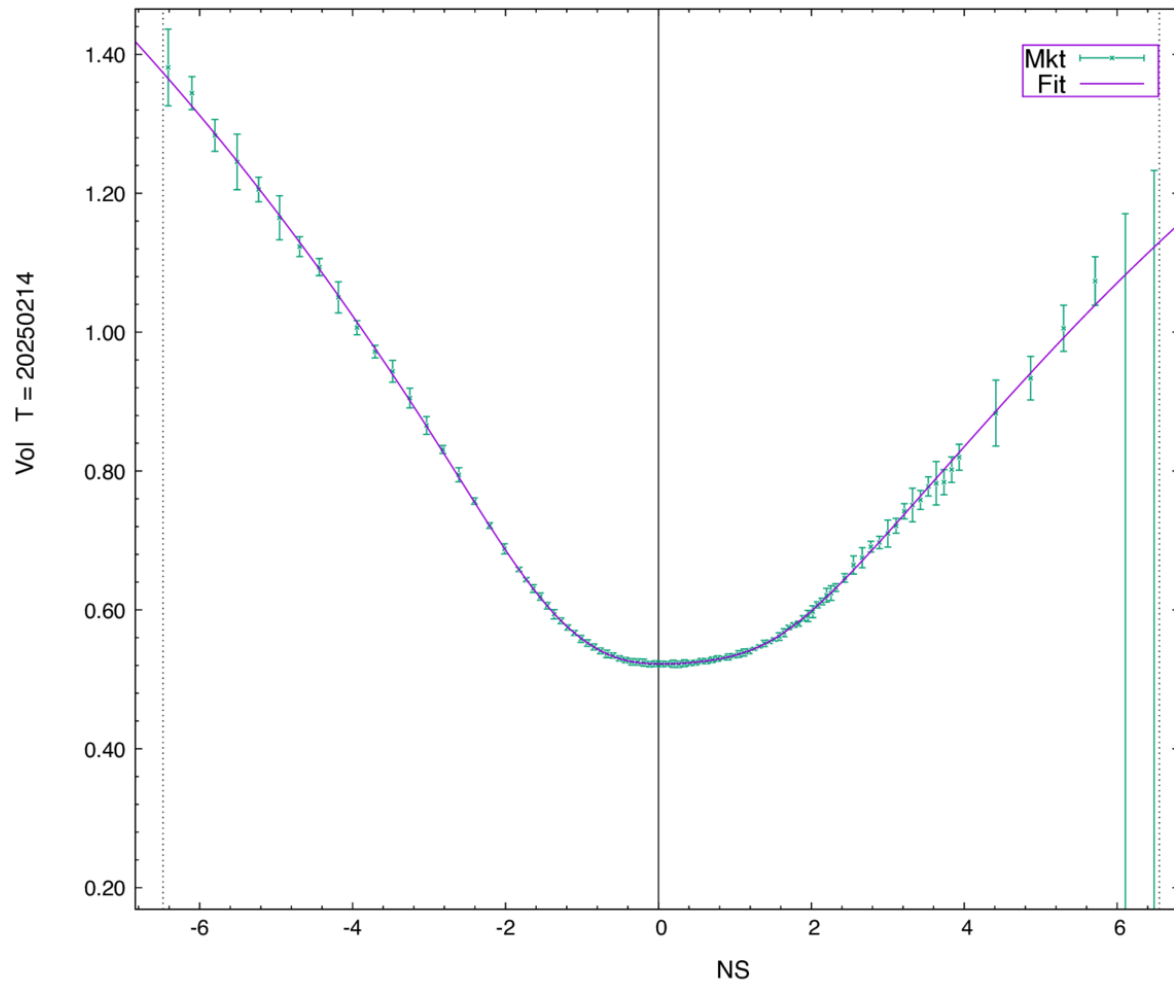


IBIT 20250128 16:00

S5, T = 2w, NS space

S5 = **SVI**

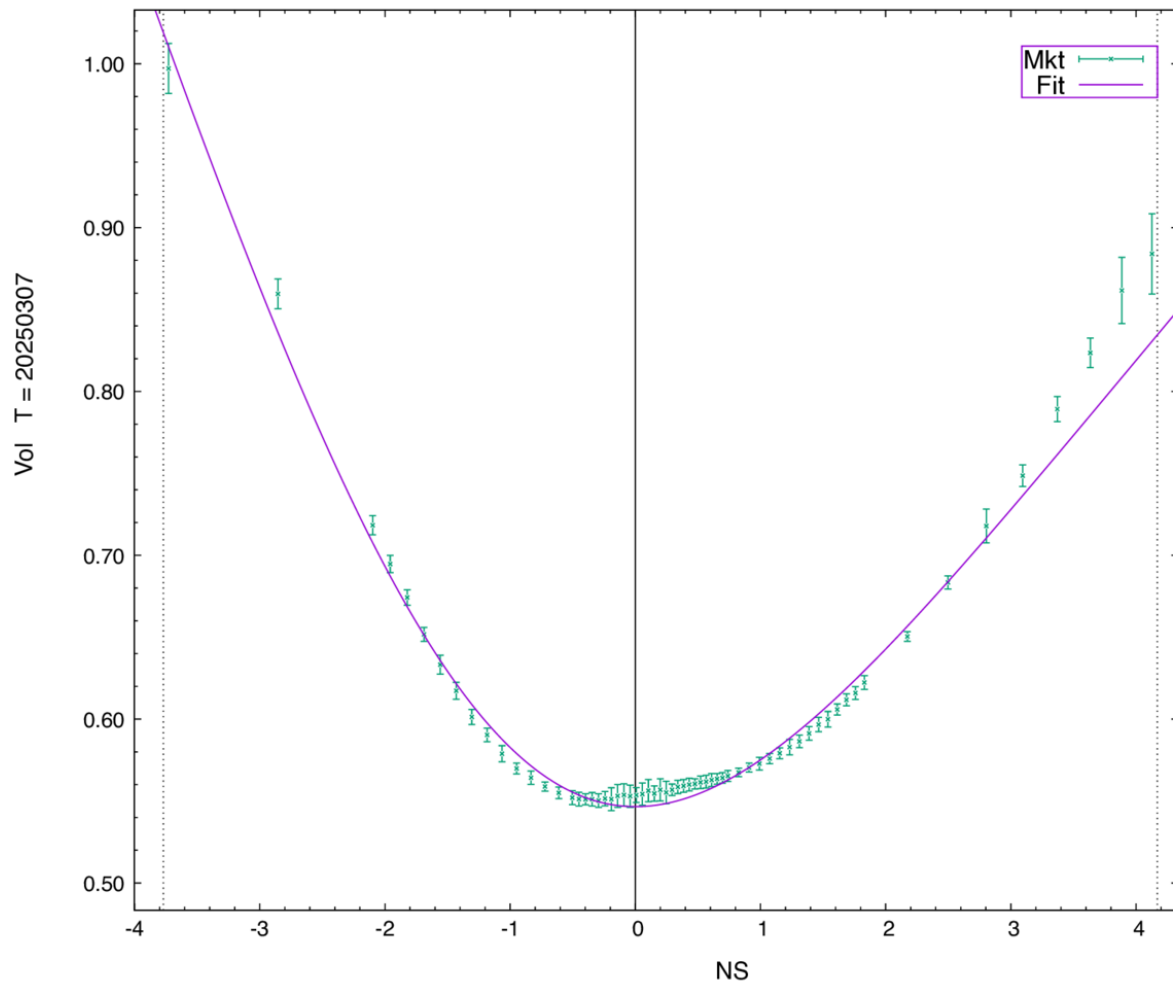
$$z := NS := \frac{\ln(K/F)}{\sigma_0 \sqrt{T}}$$



IBIT 20250128 16:00

C9w, T = 2w, NS space

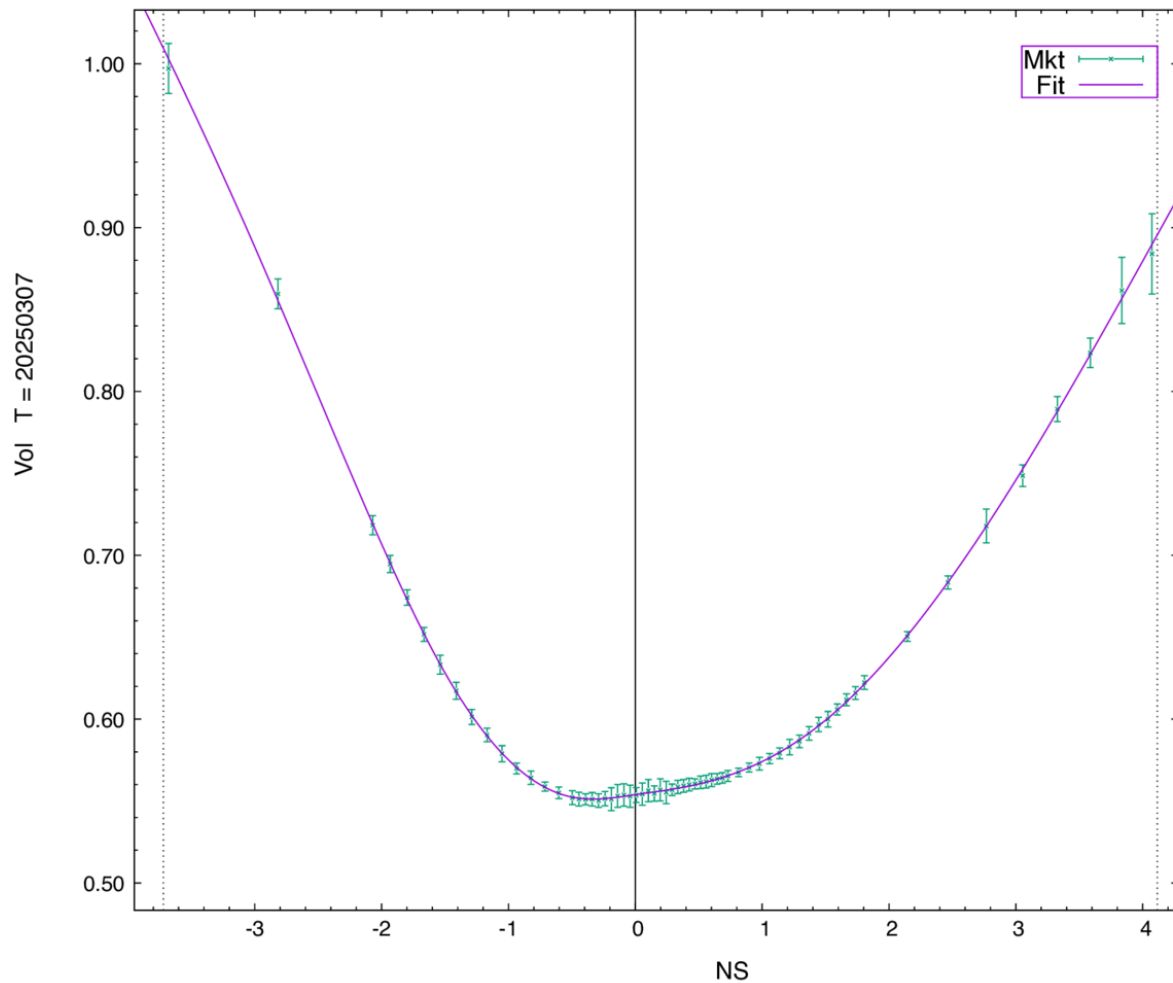
Fit is 15-20x better than SVI



IBIT 20250128 16:00

S5 T = 5w, NS space

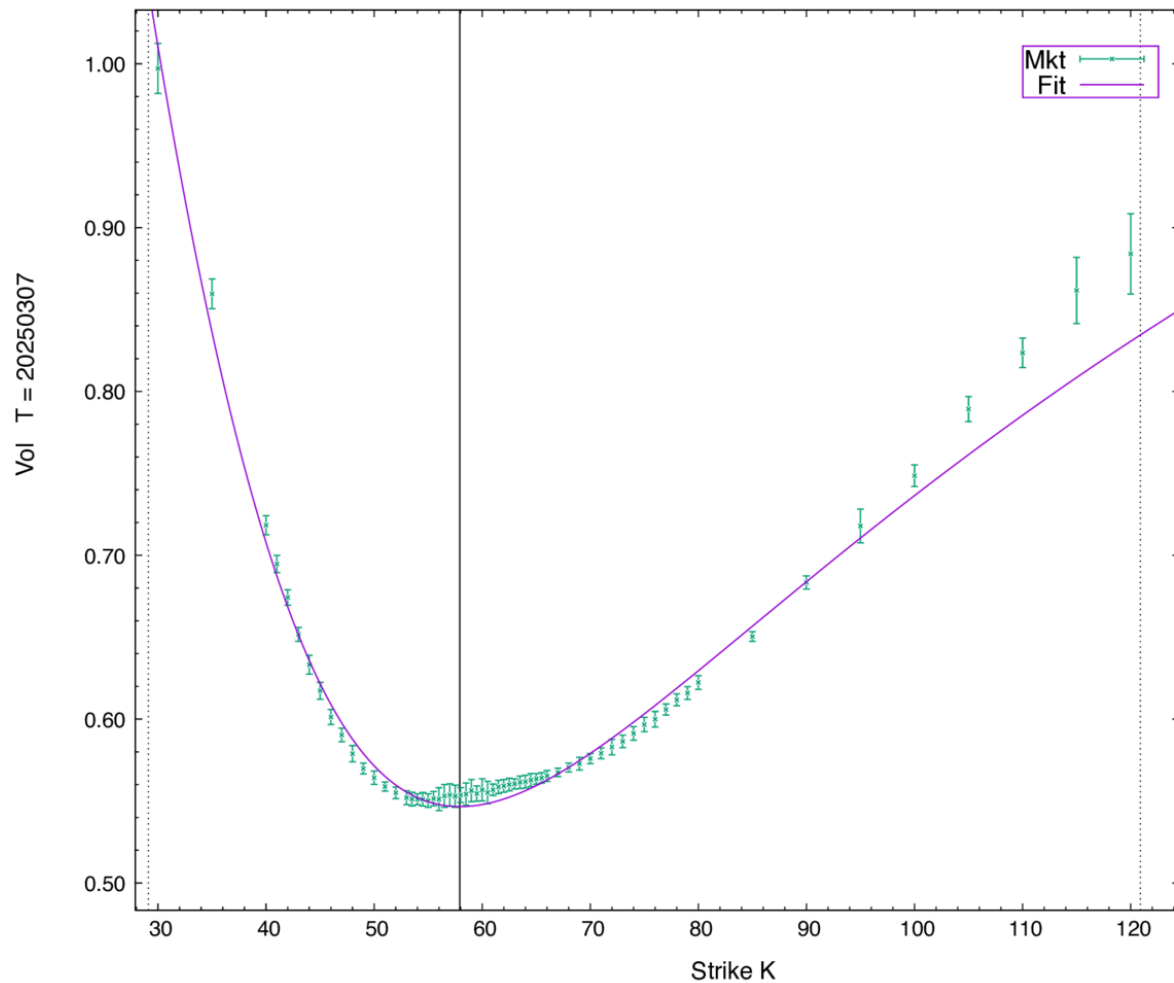
$$z := NS := \frac{\ln(K/F)}{\sigma_0 \sqrt{T}}$$



IBIT 20250128 16:00

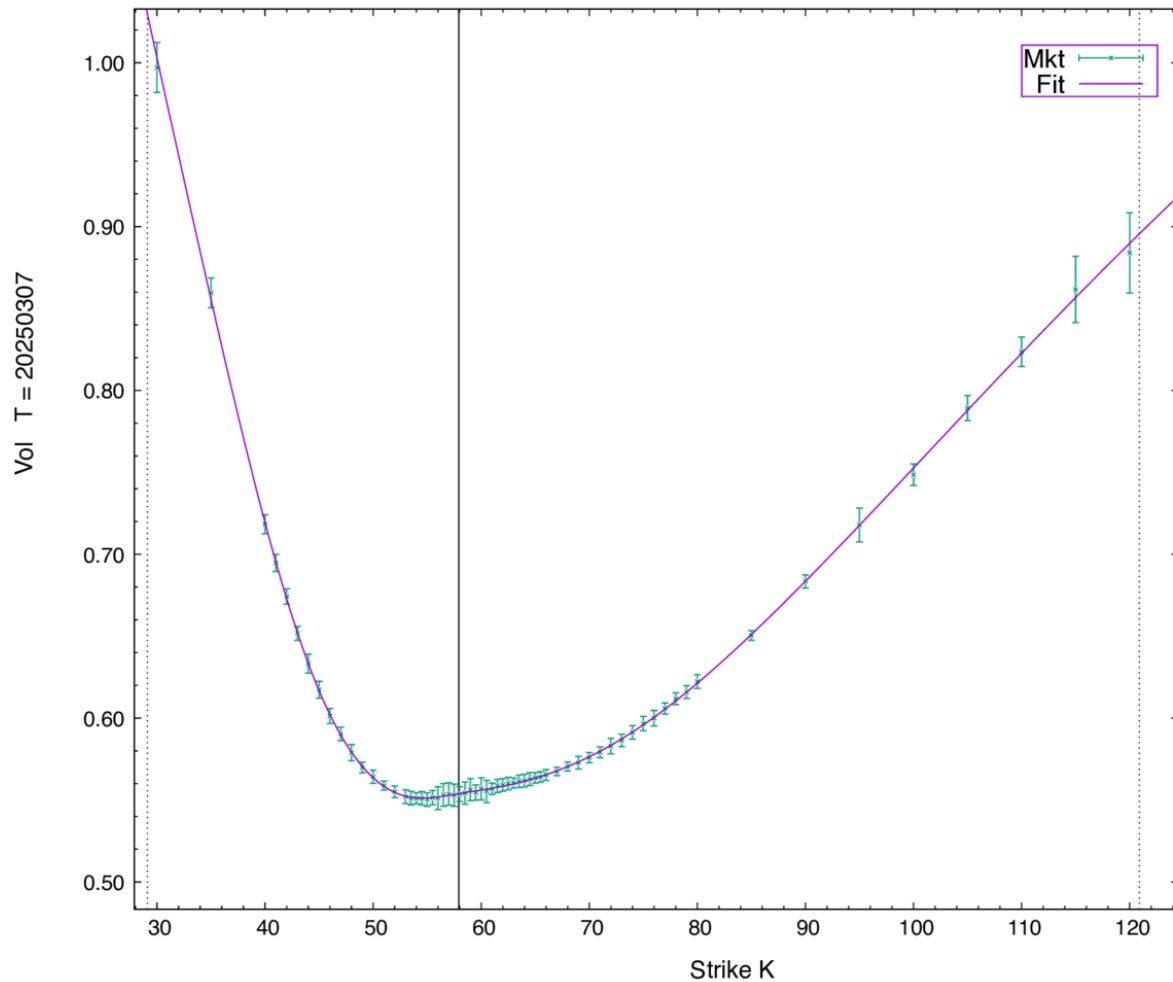
C9w, T = 5w, NS space

Fit is 13-70x better than SVI



IBIT 20250128 16:00

S5 T = 5w, K space

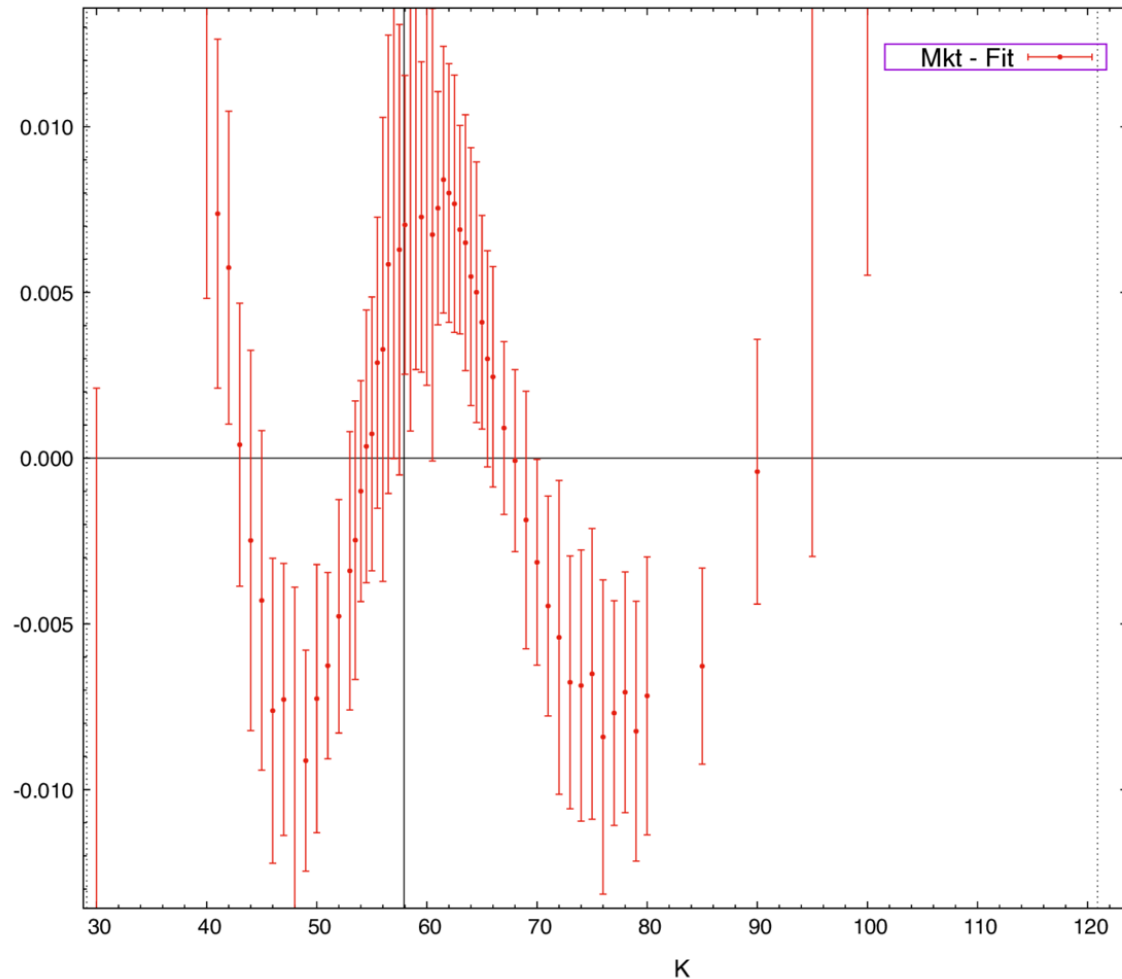


IBIT 20250128 16:00

C9w, T = 5w, K space

Fit is 13-70x better than SVI

Vol diff T = 20250307

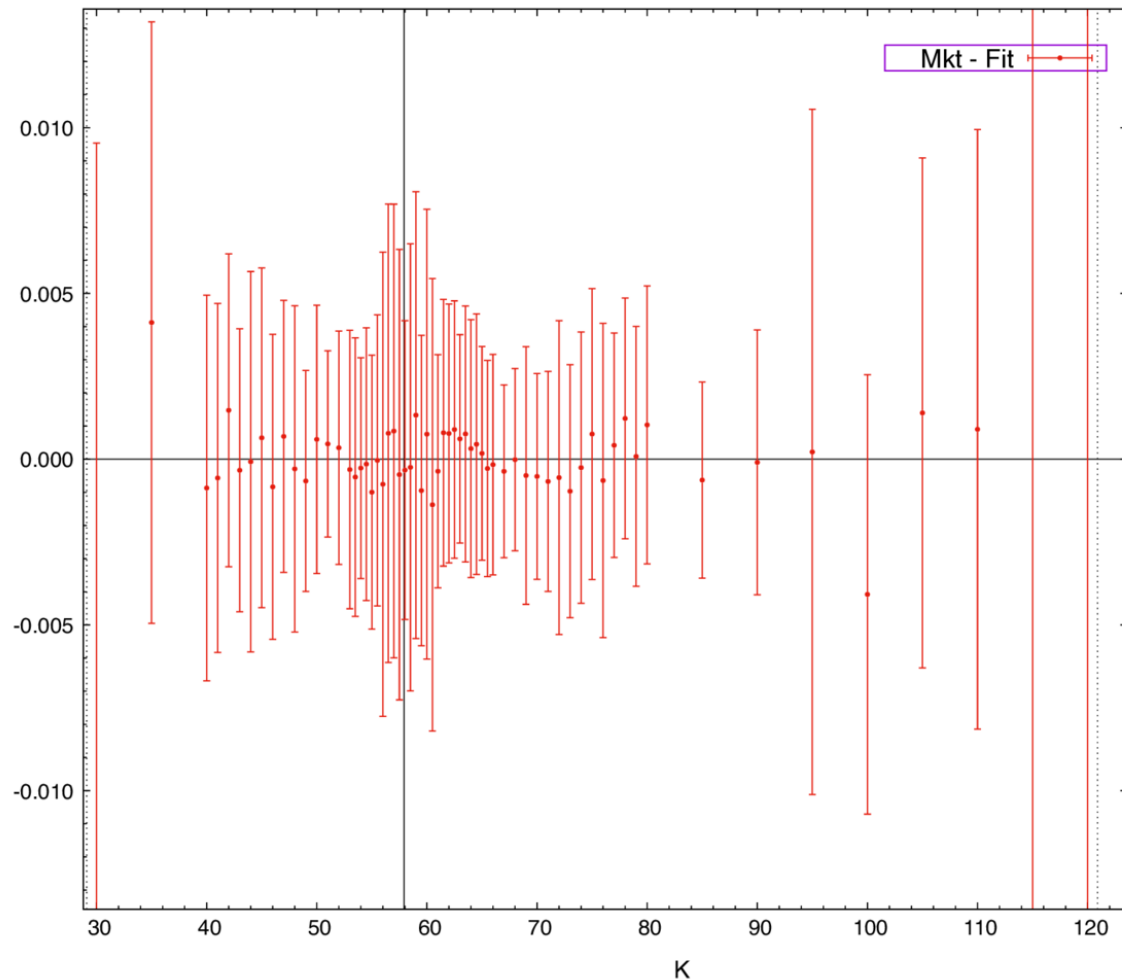


IBIT 20250128 16:00

S5 T = 5w, K space

"Vol diff" plot

Vol diff T = 20250307

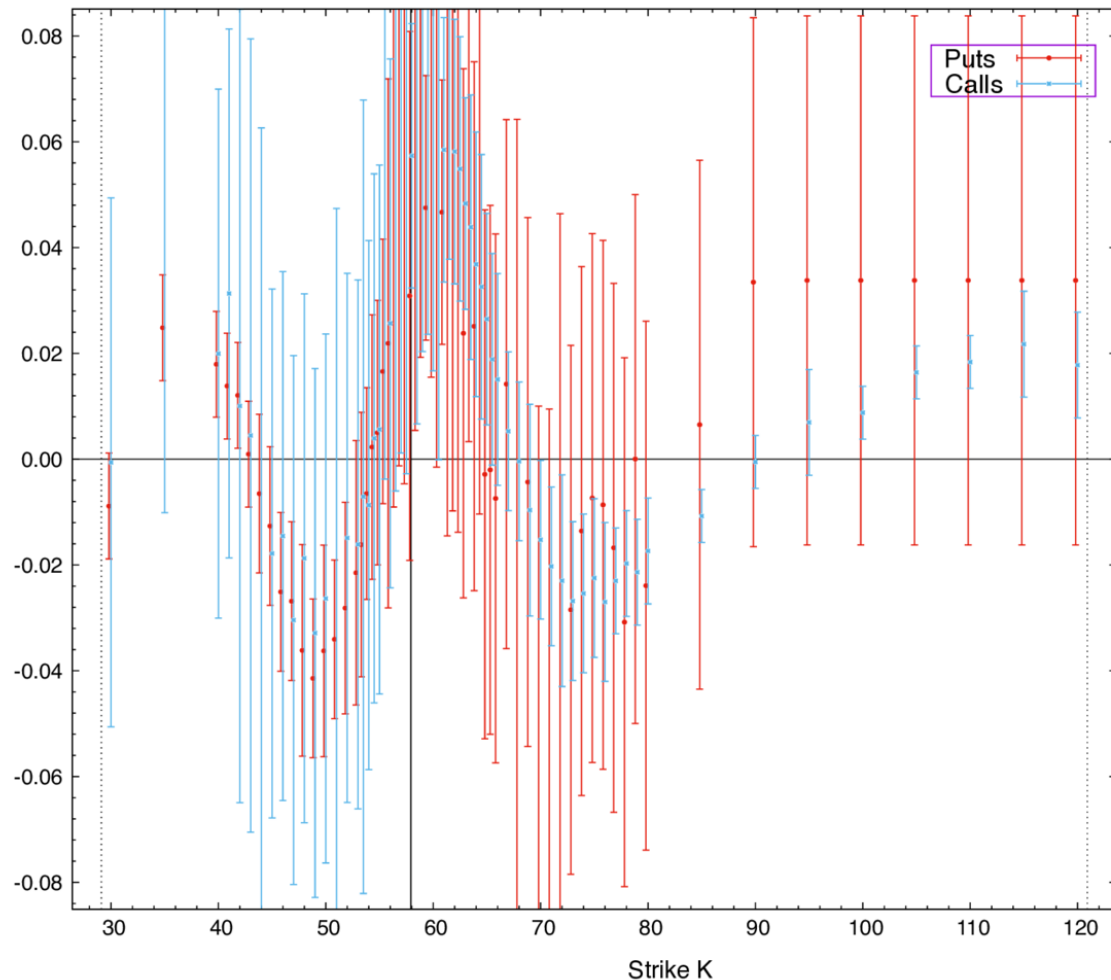


IBIT 20250128 16:00

C9w, T = 5w, K space

"Vol diff" plot

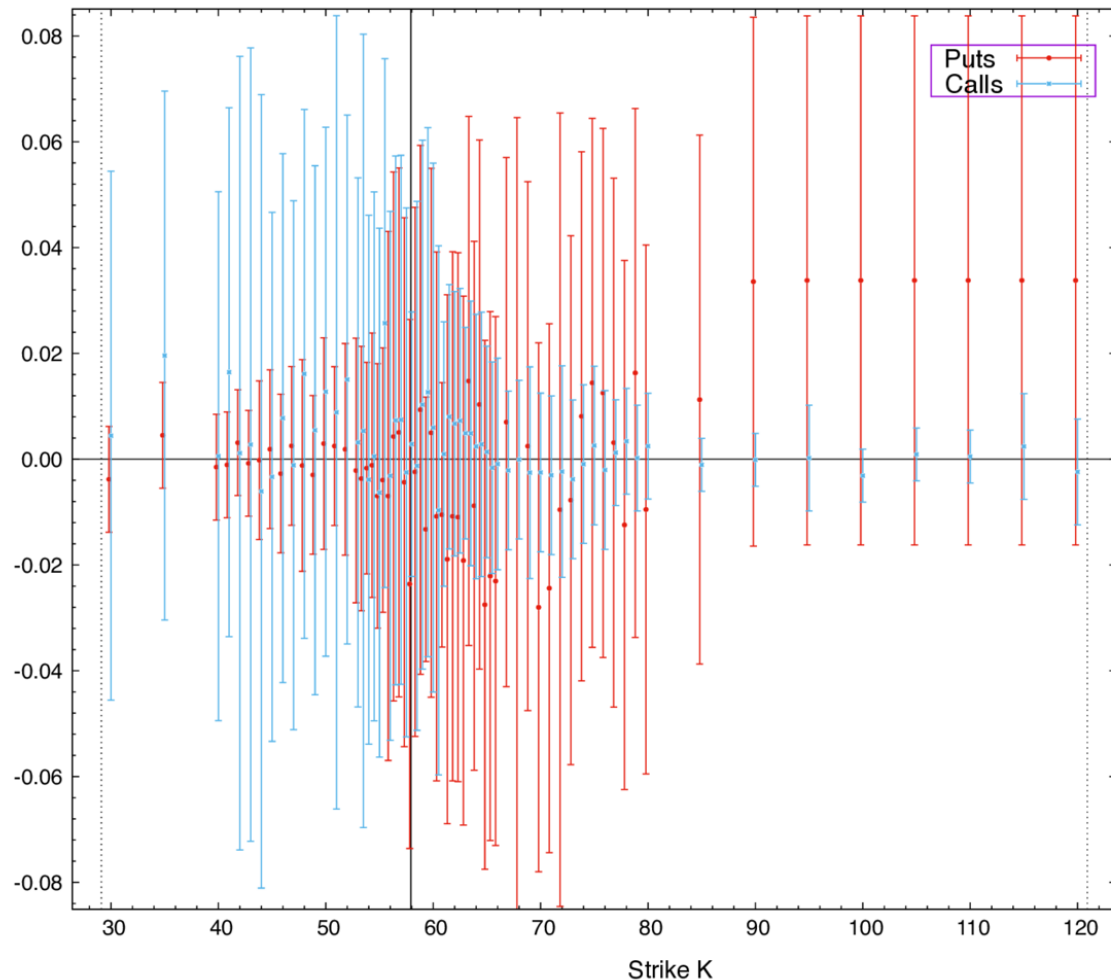
Fit is 13-70x better than SVI



IBIT 20250128 16:00

S5 T = 5w, K space

"Price diff" plot: The ultimate test!

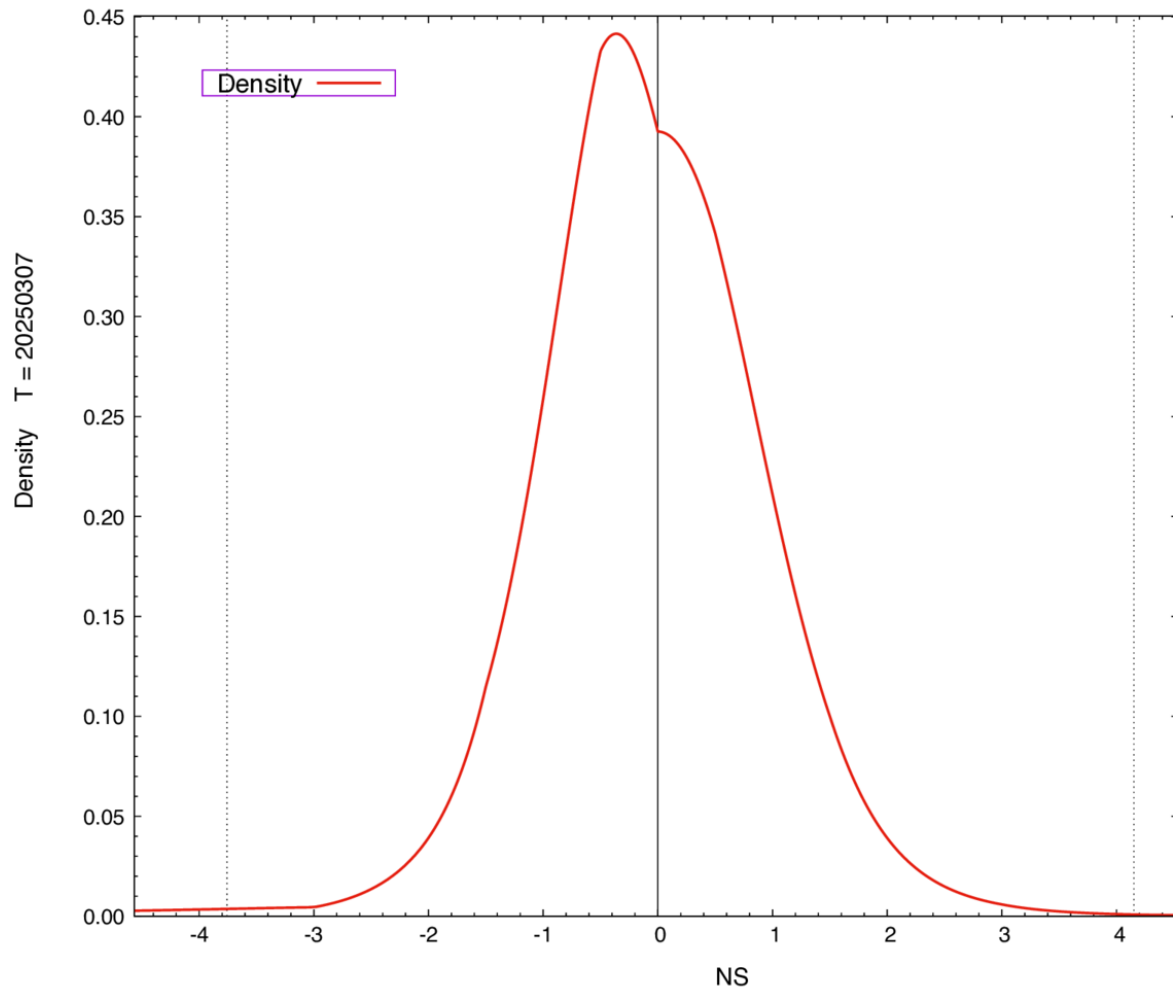


IBIT 20250128 16:00

C9w, T = 5w, K space

"Price diff" plot: The ultimate test!

Fit is 13-70x better than SVI

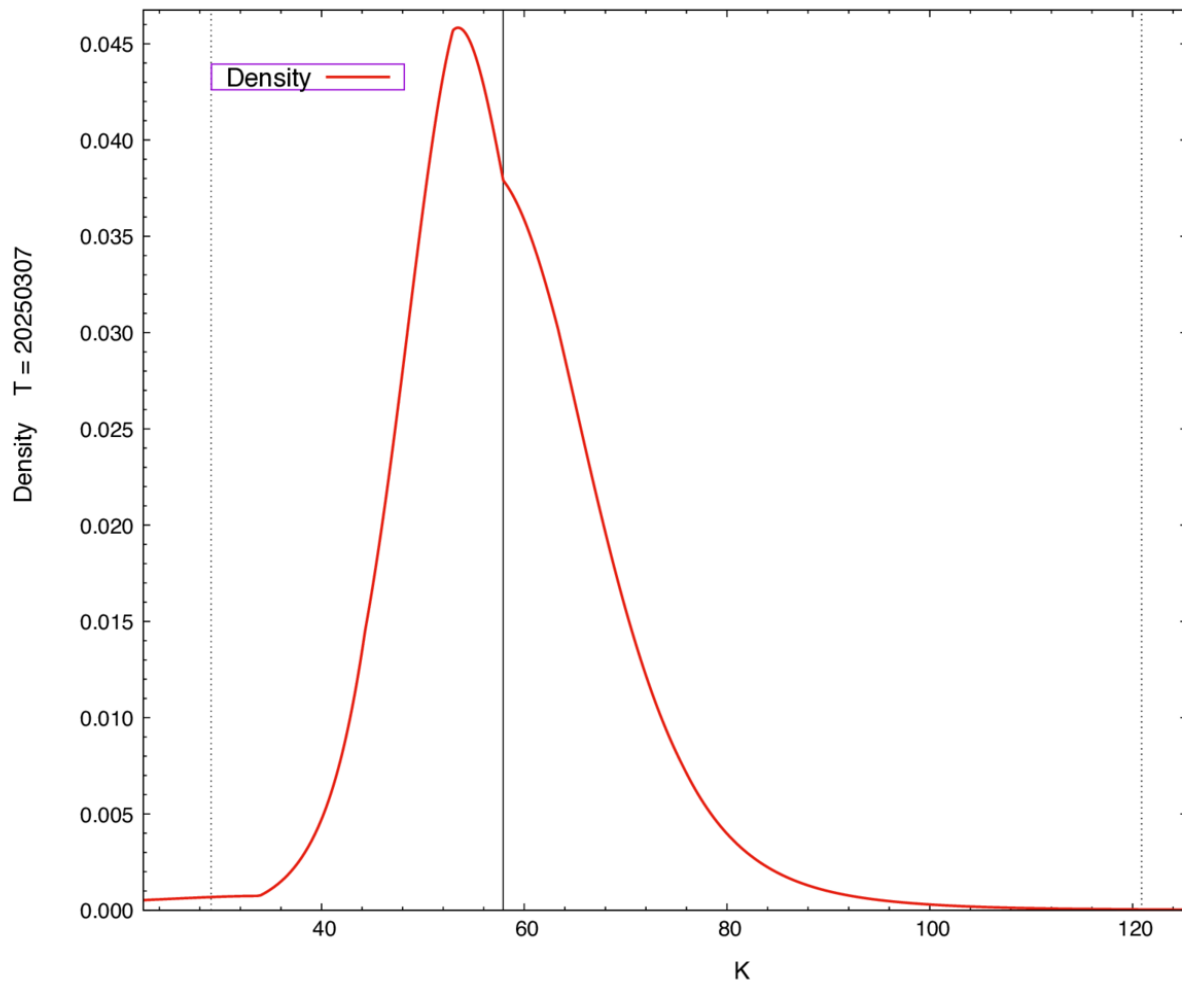


IBIT 20250128 16:00

C9w, T = 5w, NS space

Implied density

A pretty fat put wing...

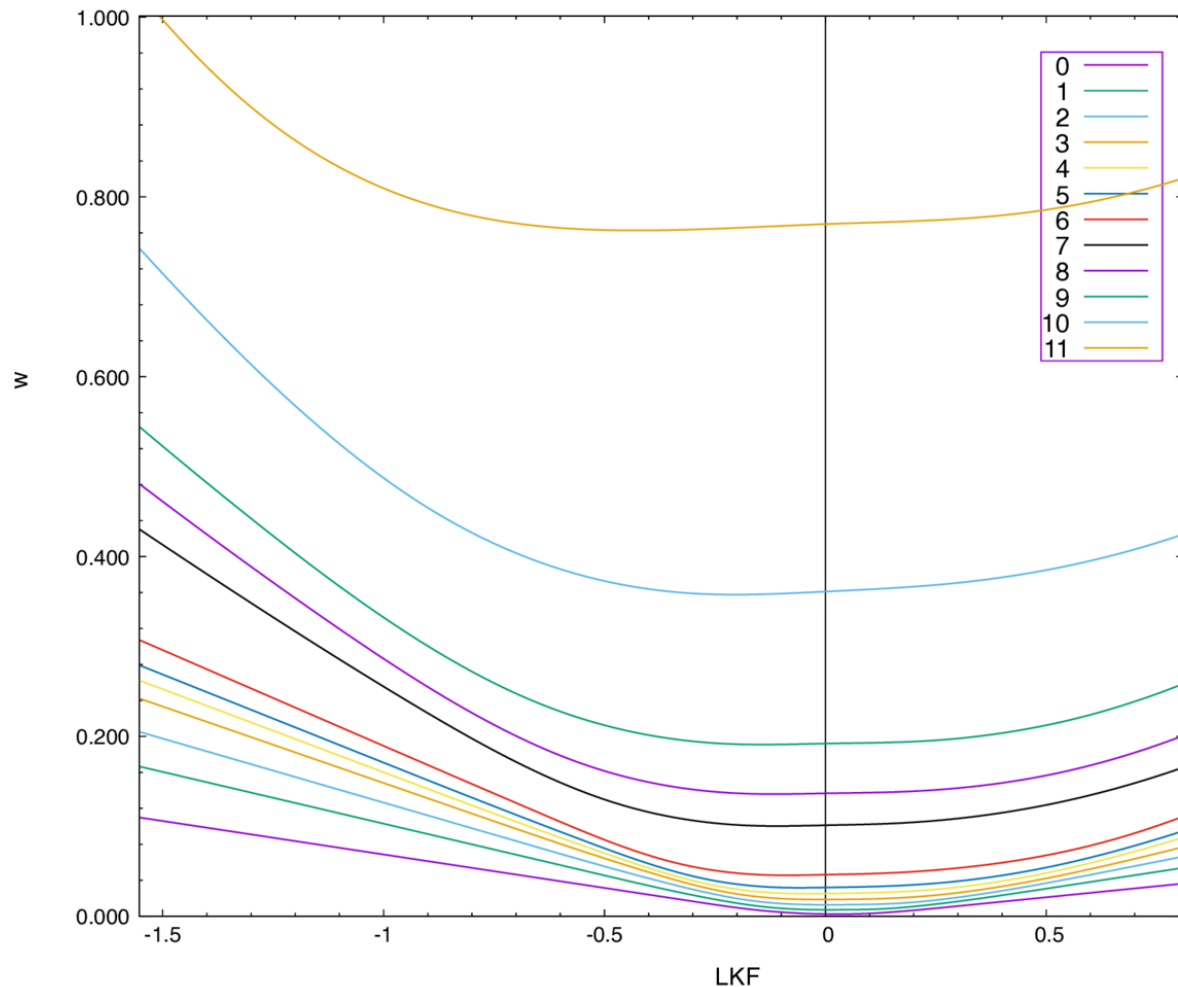


IBIT 20250128 16:00

C9w, T = 5w, K space

Implied density

A pretty fat put wing...

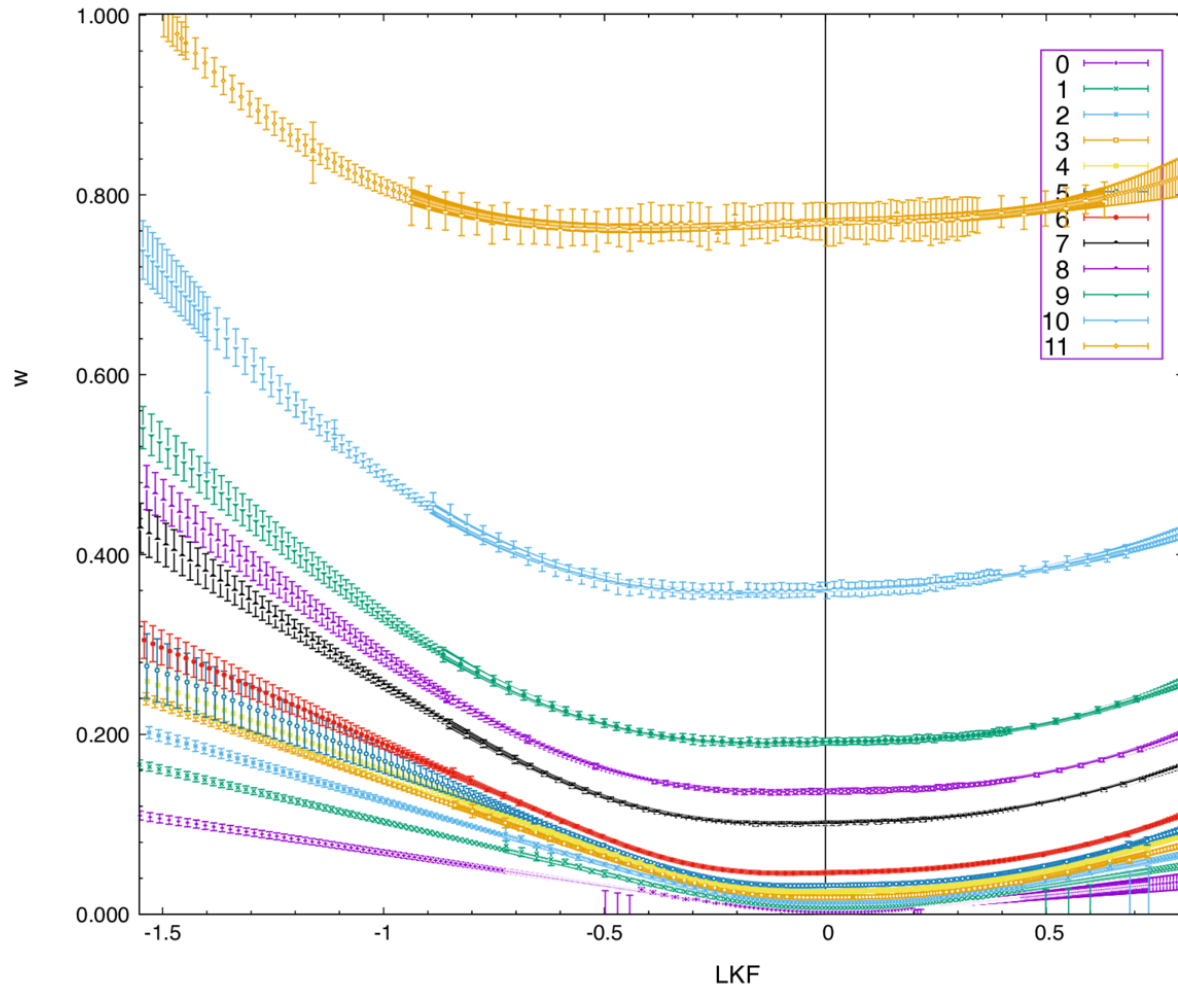


IBIT 20250128 16:00

Total Variance plot

No calendar arbitrage!

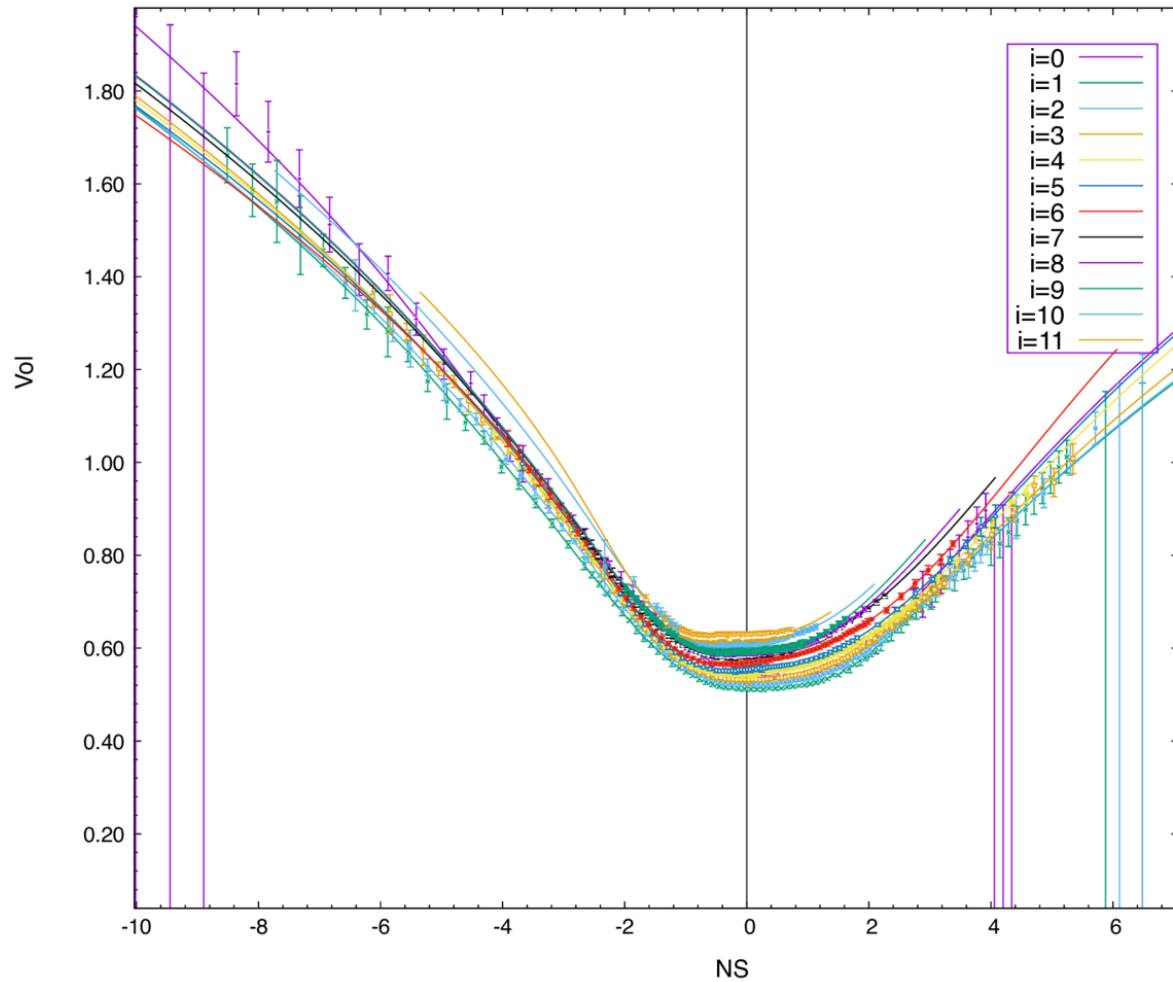
$$w(T, K) := \sigma(T, K)^2 T$$



IBIT 20250128 16:00

Total Variance plot

With error bars!

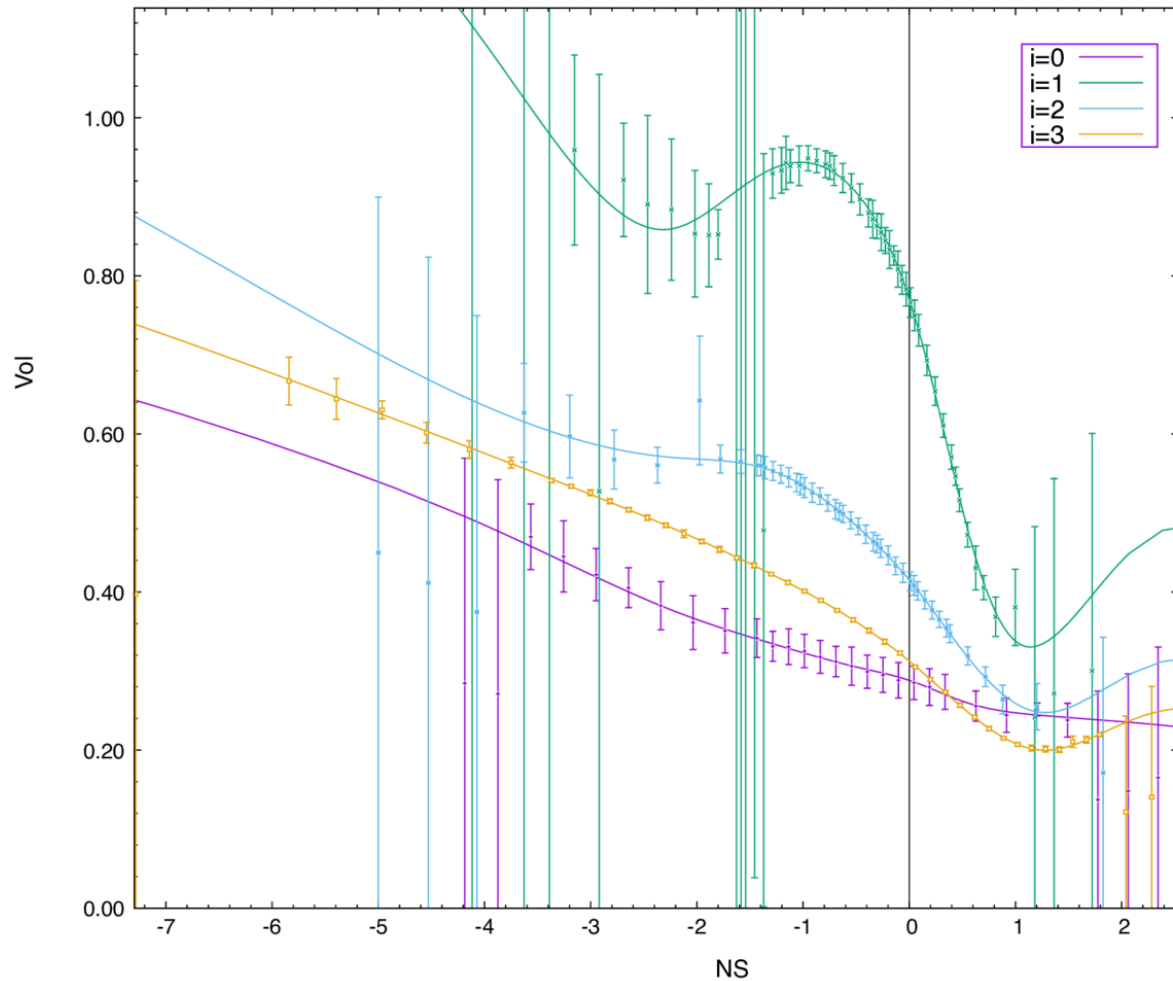


IBIT 20250128 16:00

All vols by NS

IBIT options: Conclusions

- Implied borrows can be very different from actual (overnight) borrow fees charged by agent banks/prime brokers.
- As an options market becomes liquid, SVI or other simple curves will very quickly not be flexible enough to match the market in a bias-free manner.

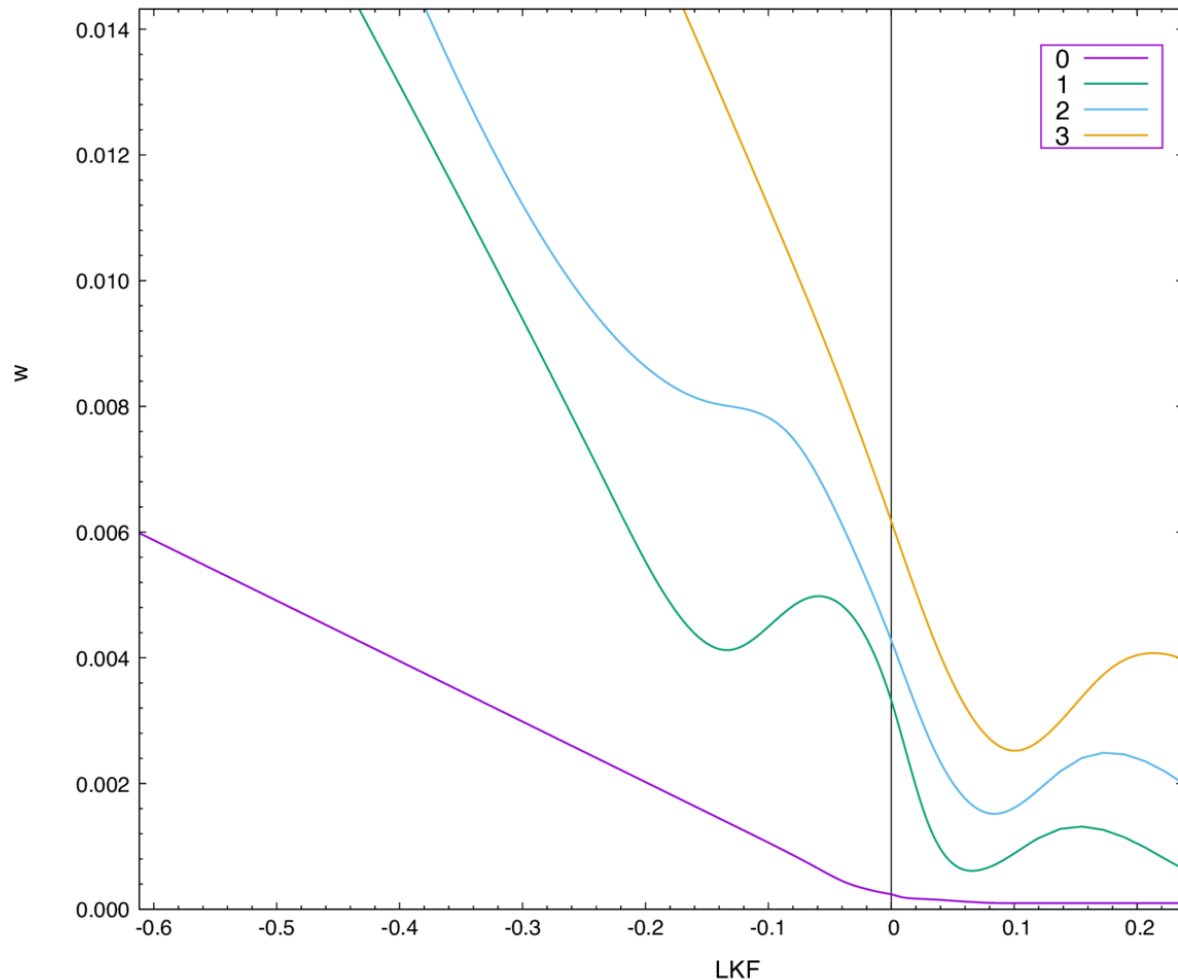


AEX 2016-06-22

Day before **Brexit!**

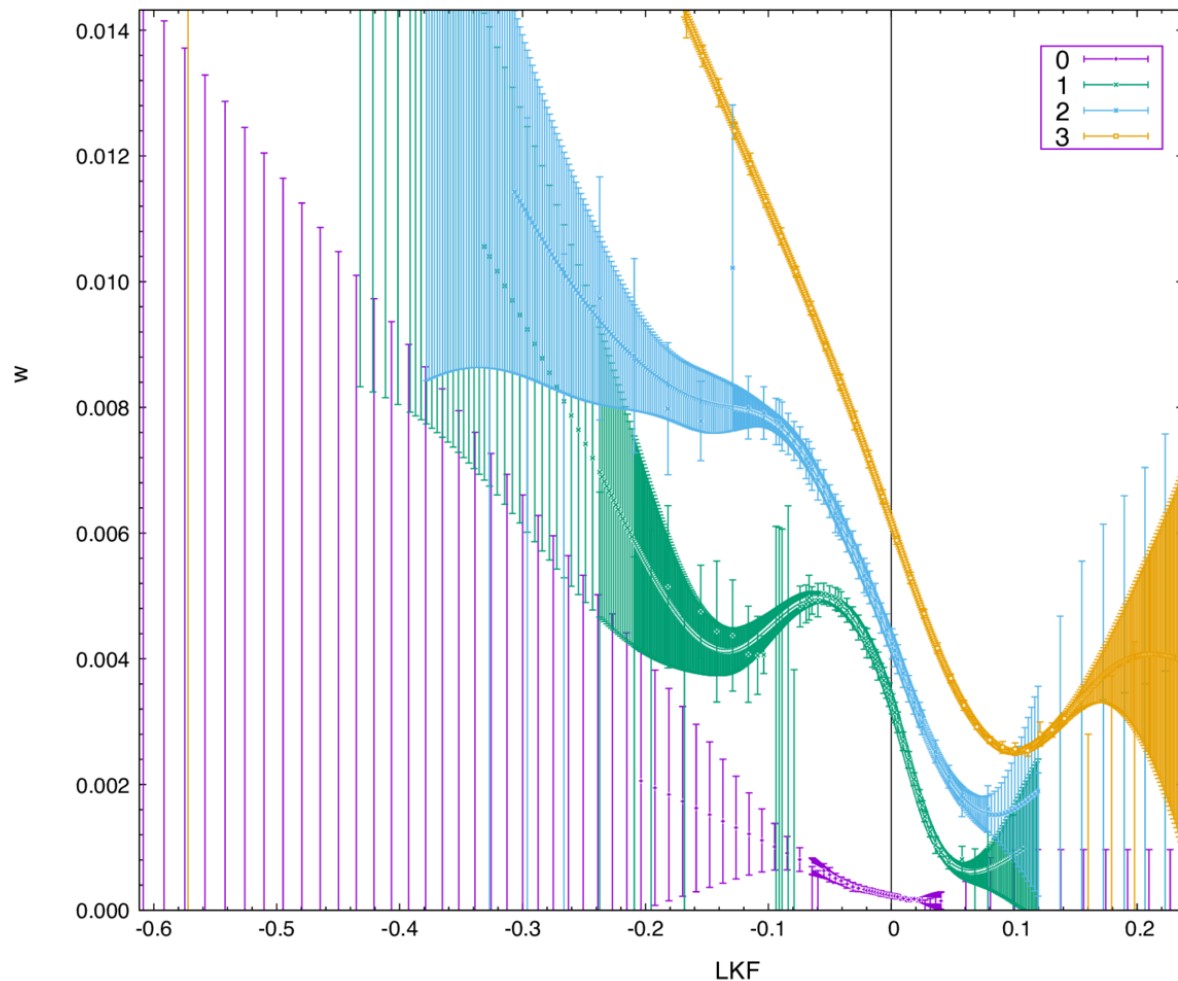
Vol vs NS

$$z := \text{NS} := \frac{\ln(K/F)}{\sigma_0 \sqrt{T}}$$



Fitting **AEX** on day
before **Brexit**

Total Var plot



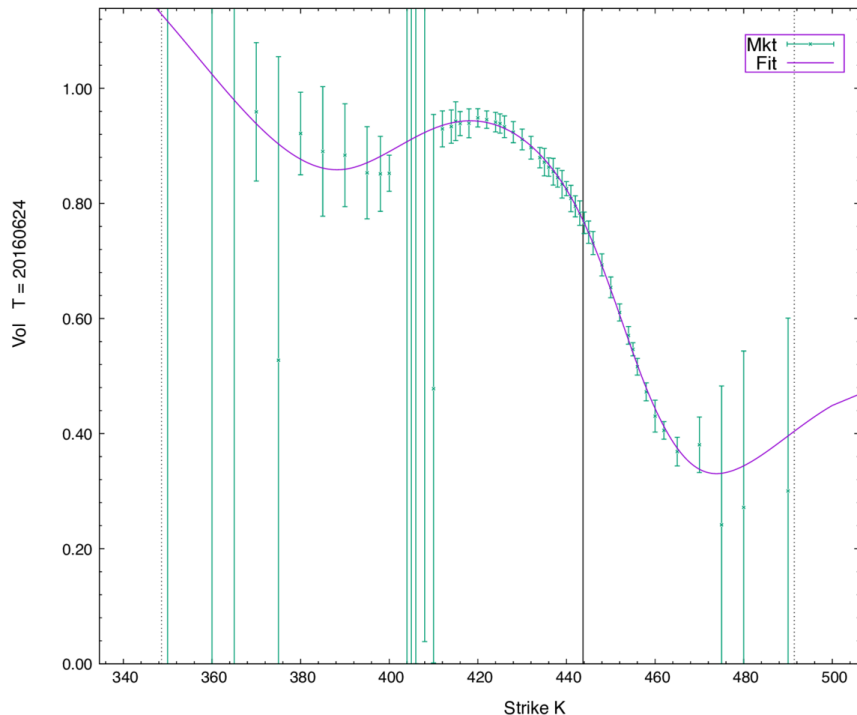
Fitting **AEX** on day
before **Brexit**

Total Var plot
with error bars

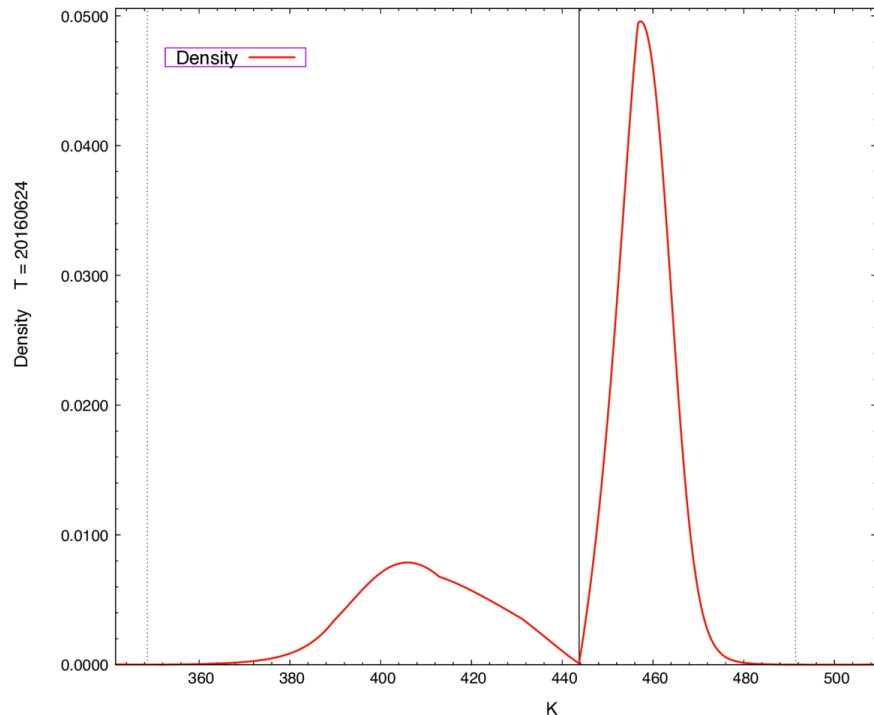
AEX on day before **Brexit** vote:

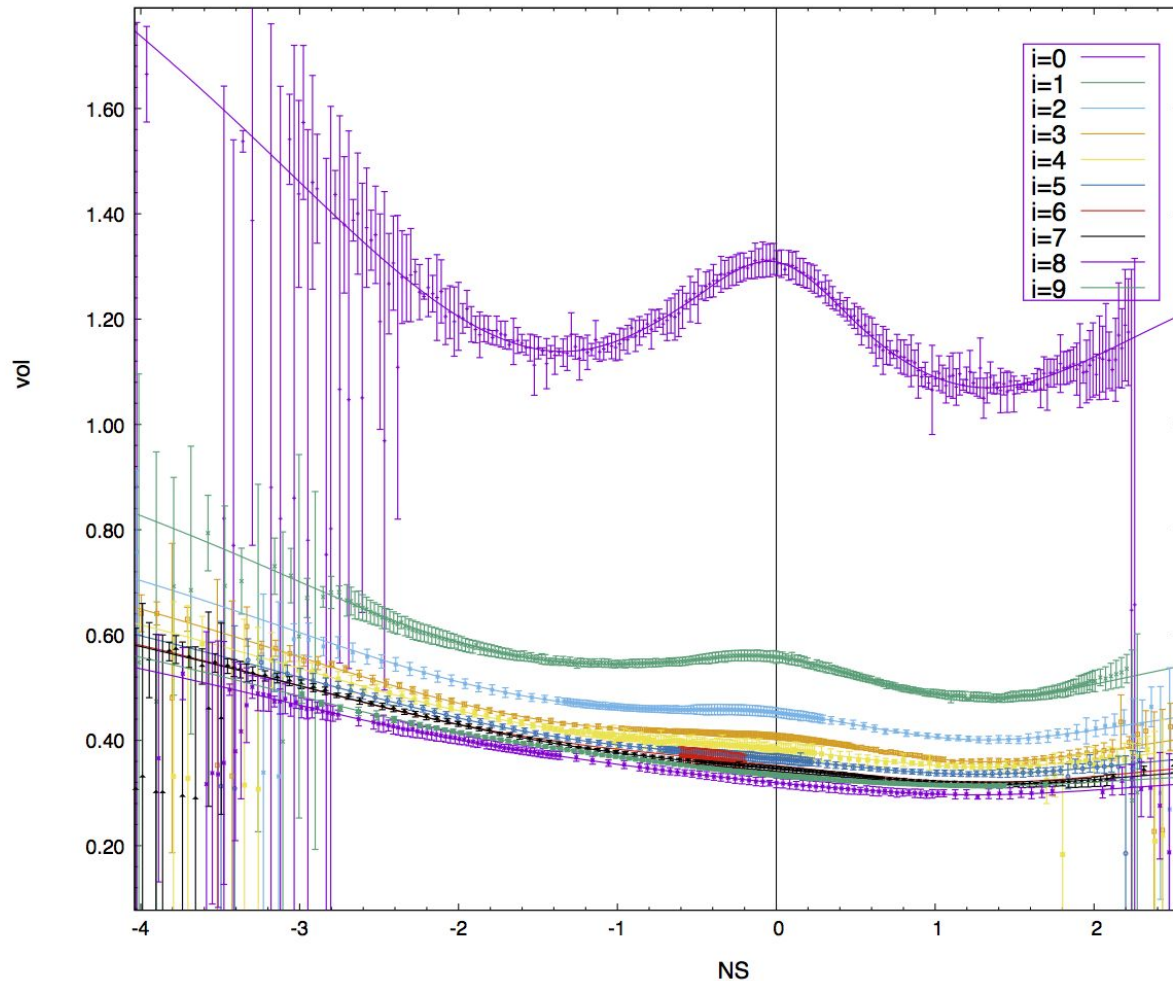
T=2d, vols and implied density (AEX dropped 5.9%)

AEX 20160622-160000 C10w: T=0.0056, i=1, chi=0.115, avE5=14.1



AEX 20160622-160000 C10w: T=0.0056, i=1, chi=0.115, avE5=14.1





AMZN 2018-04-26
earnings day

C8 Vol vs NS

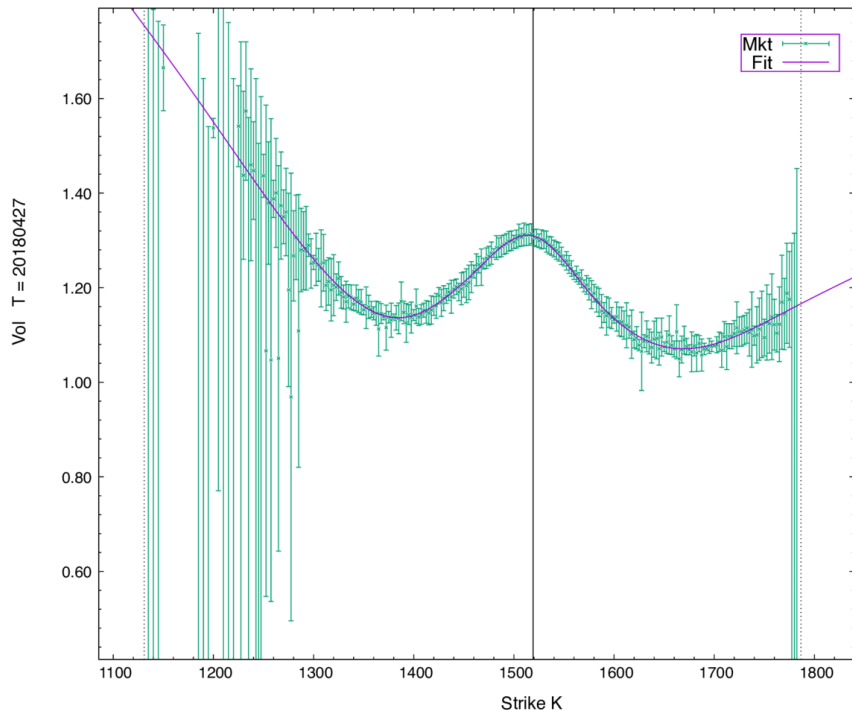
Interesting Thursday: Earnings, **new**
weekly listed (only a few strikes in PW)

$$z := NS := \frac{\ln(K/F)}{\sigma_0 \sqrt{T}}$$

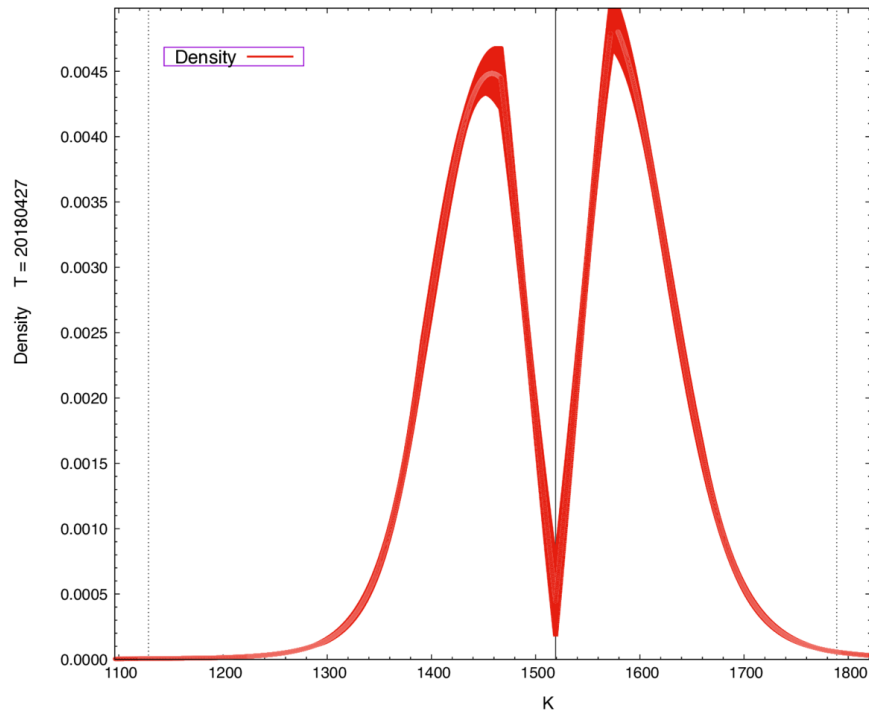
AMZN 2018-04-26 earnings day:

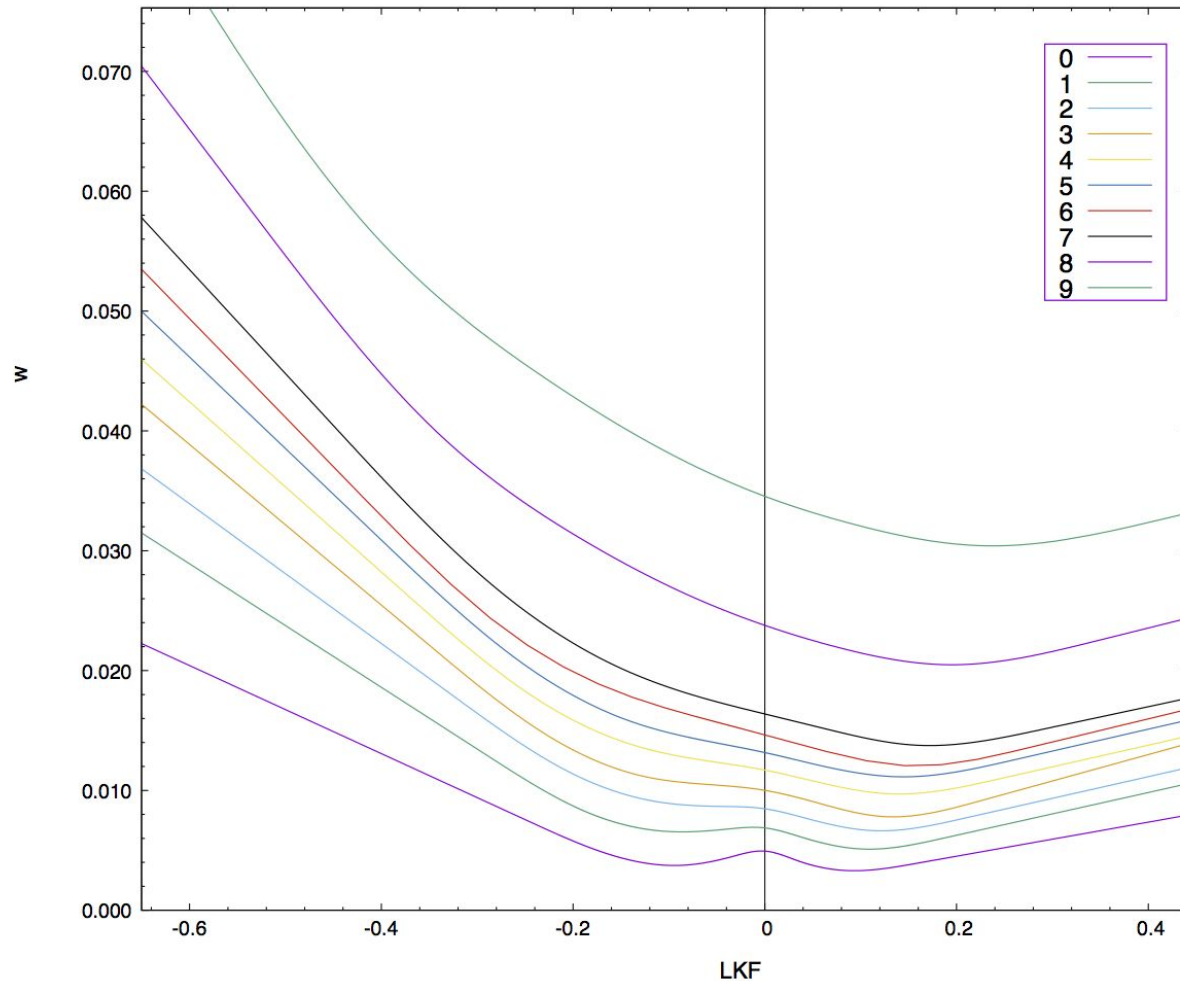
T=1d, vols and implied density — most bimodal density ever!

AMZN 20180426-154500 C8: T=0.0029, i=0, chi=0.112, avE5=19.4



AMZN 20180426-154500 C8: T=0.0029, i=0, chi=0.112, avE5=19.4





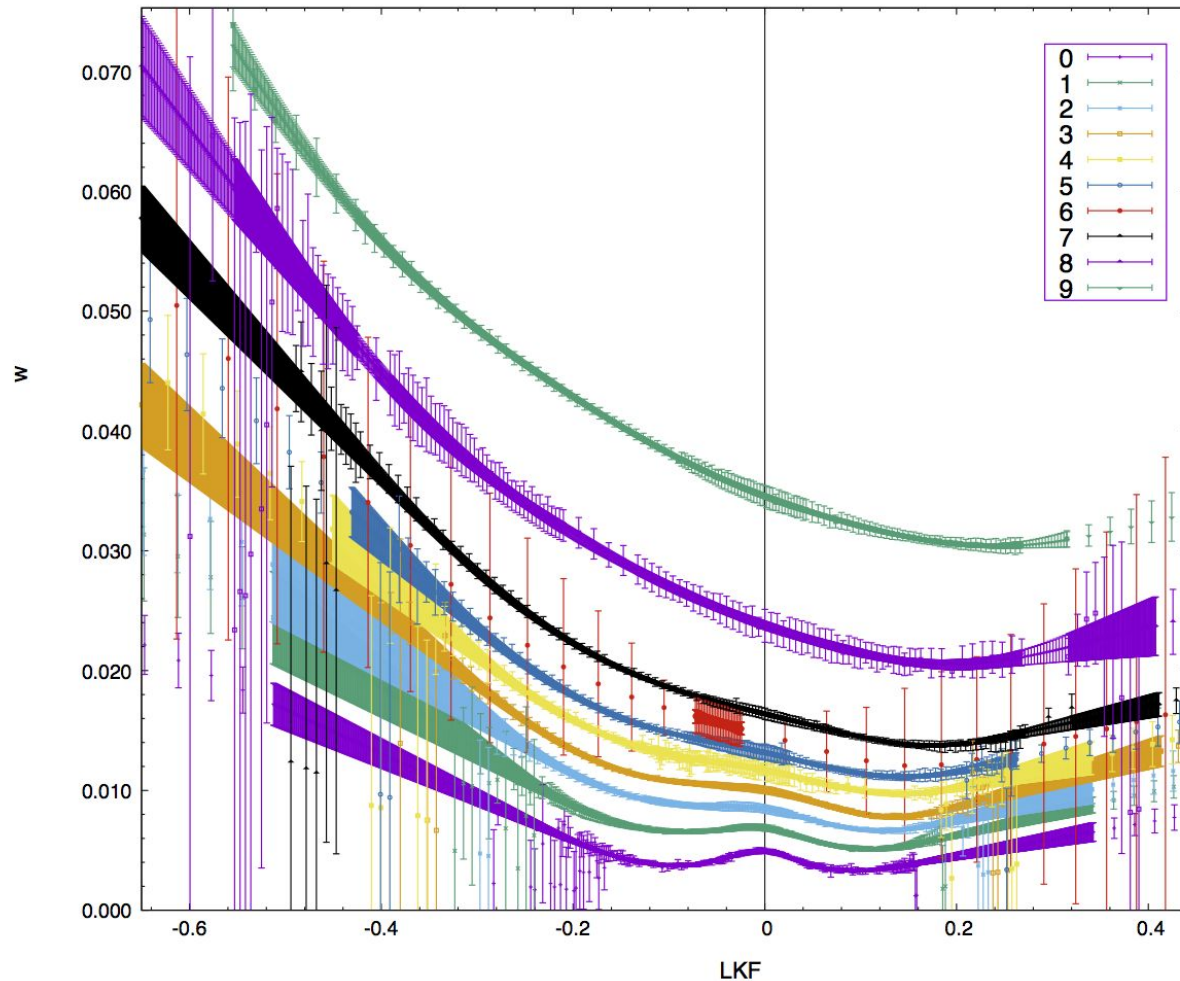
AMZN 2018-04-26
earnings day

C8 **total variance** plot

First 10 terms

No calendar arbitrage! (Or butterfly...)

Interesting Thursday: Earnings, new weekly listed (**i=6**), etc.

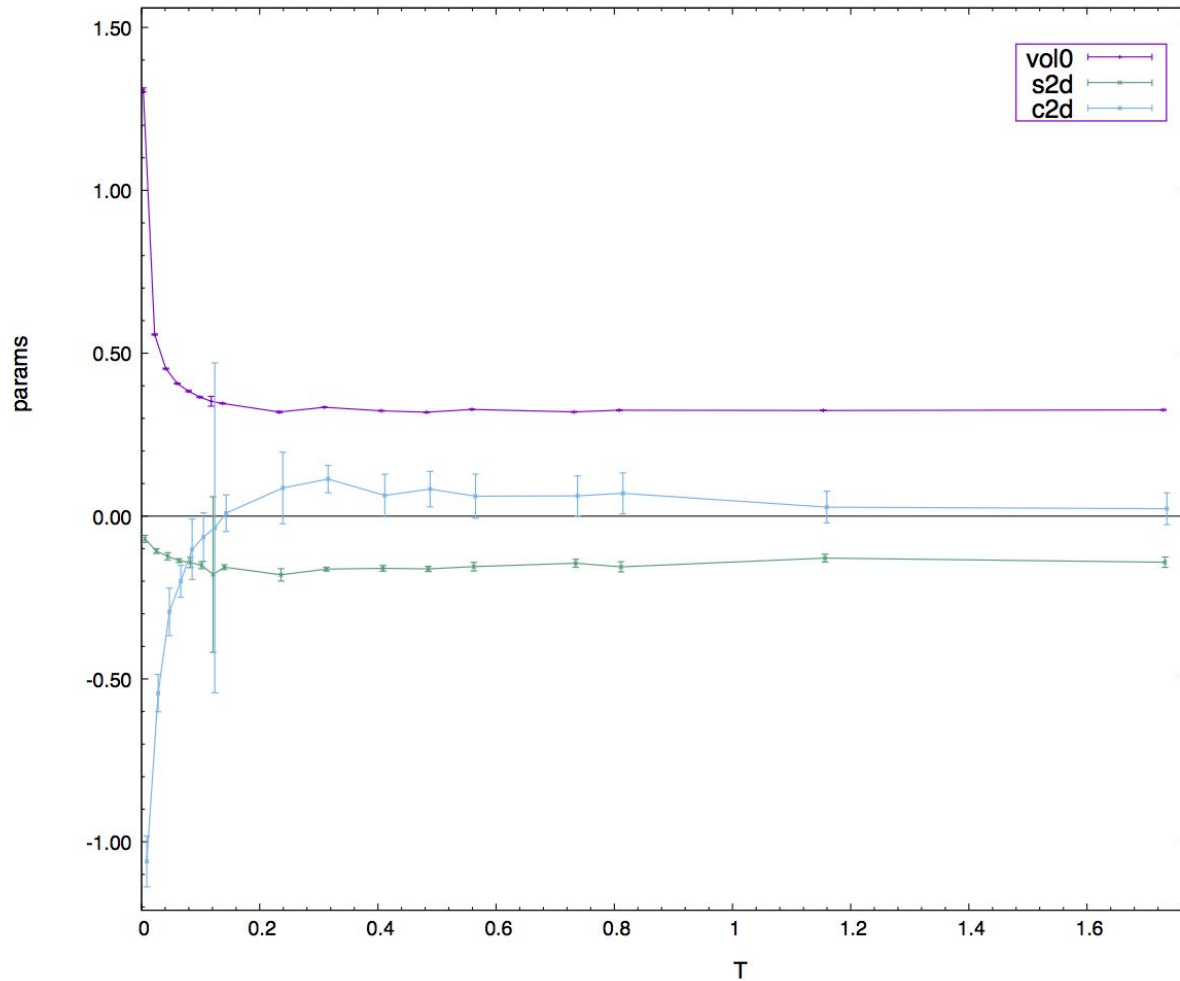


AMZN 2018-04-26
earnings day

C8 **total variance** plot

First 10 terms, with **errors bars**

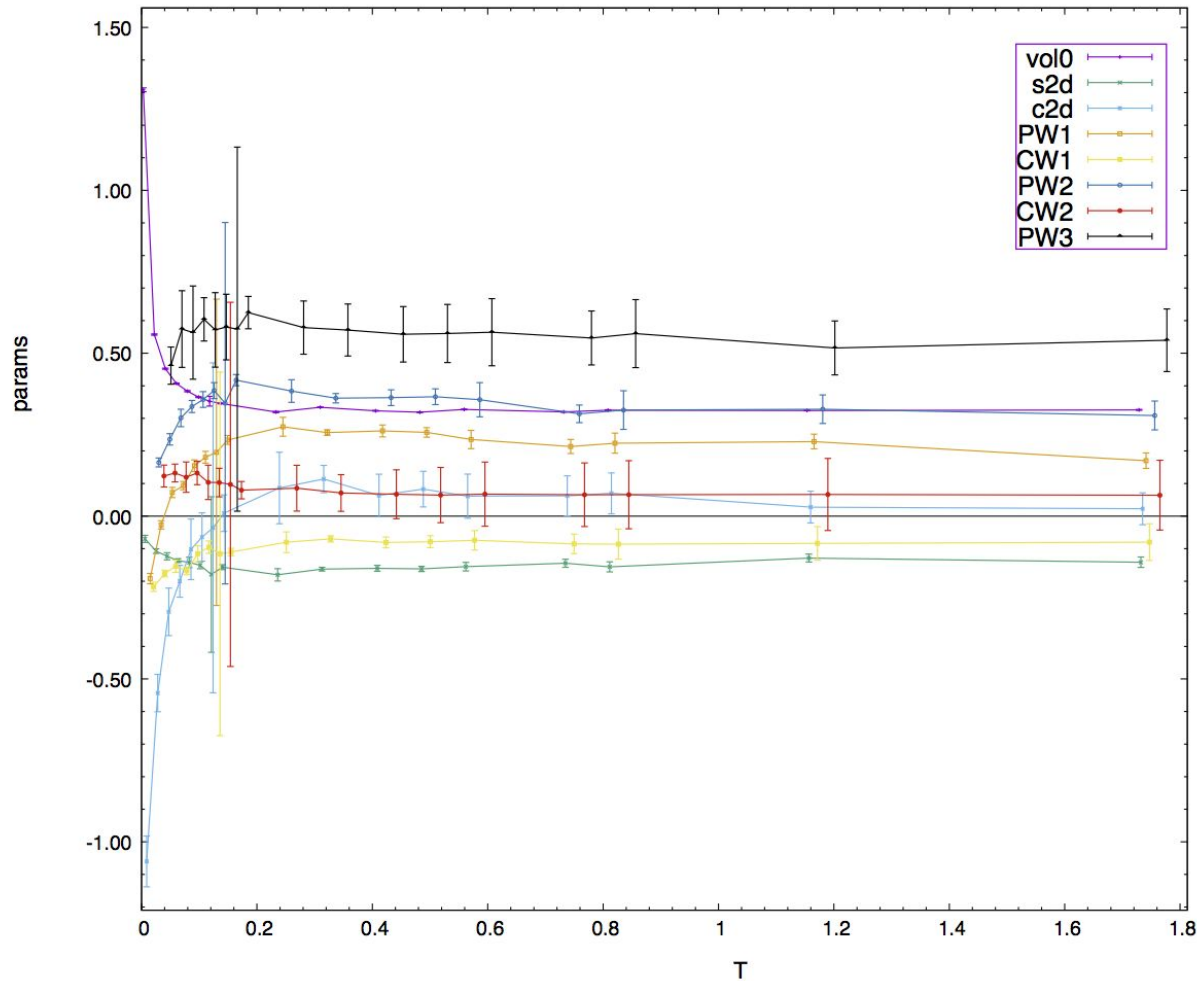
Interesting Thursday: Earnings, new weekly
listed (**i=6**), etc.



AMZN 2018-04-26
earnings day

C8 parameter term-structure
First 3: vol0, s2, c2

Essentially flat shape params after 3m



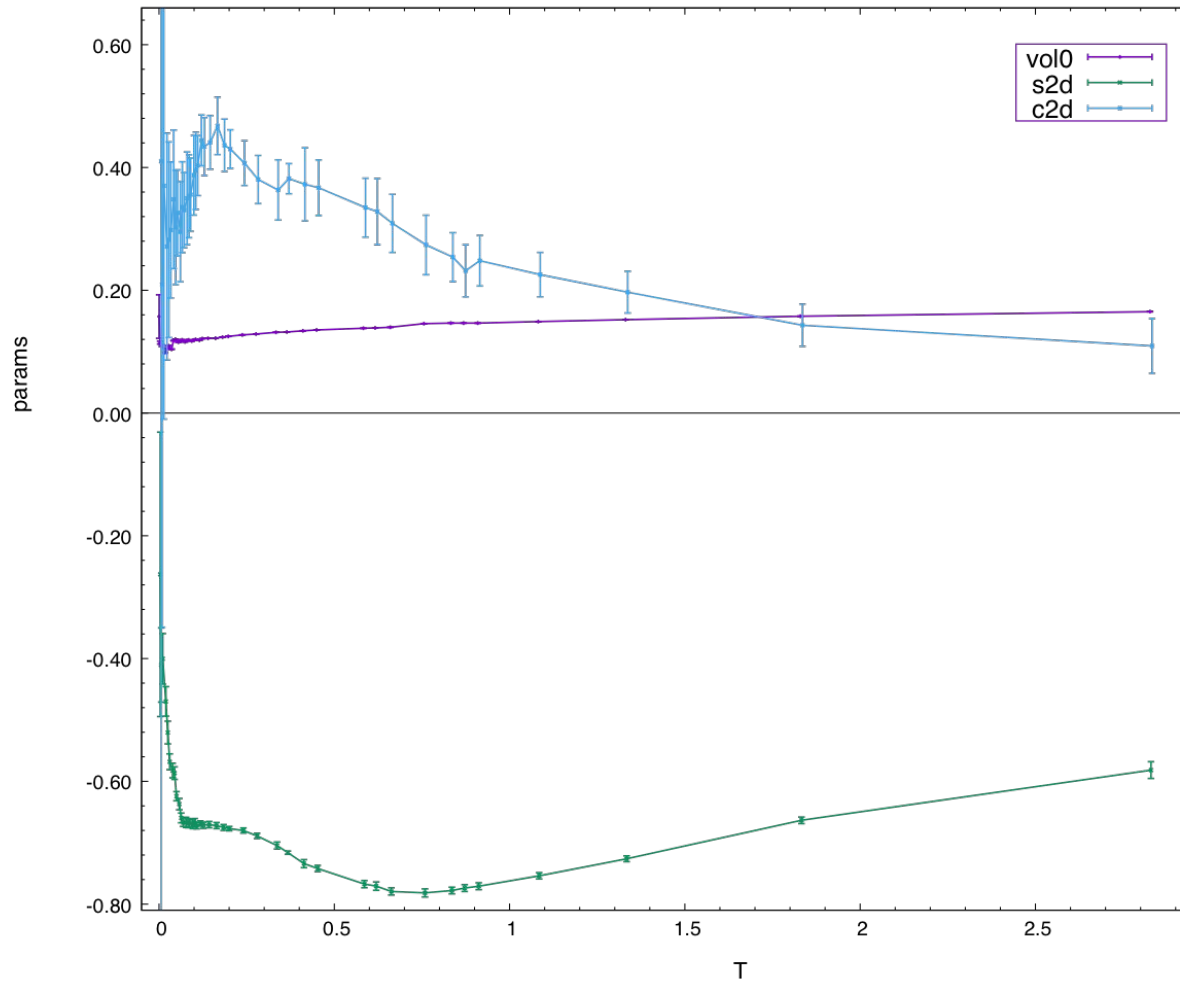
AMZN 2018-04-26
earnings day

C8 parameter term-structure

Essentially flat shape params after 3m

SPX during the covid crash of March 2020

- SPX vol curves and surfaces had unprecedented shapes.
- Supposedly even some Tier 1 bank(s) didn't manage to produce a tradable SPX surface for 2 days (e.g. arbitrage-free).
- But the options market functioned perfectly fine at all times and those “funky” shapes reflected quite precise & consistent forward-looking views.



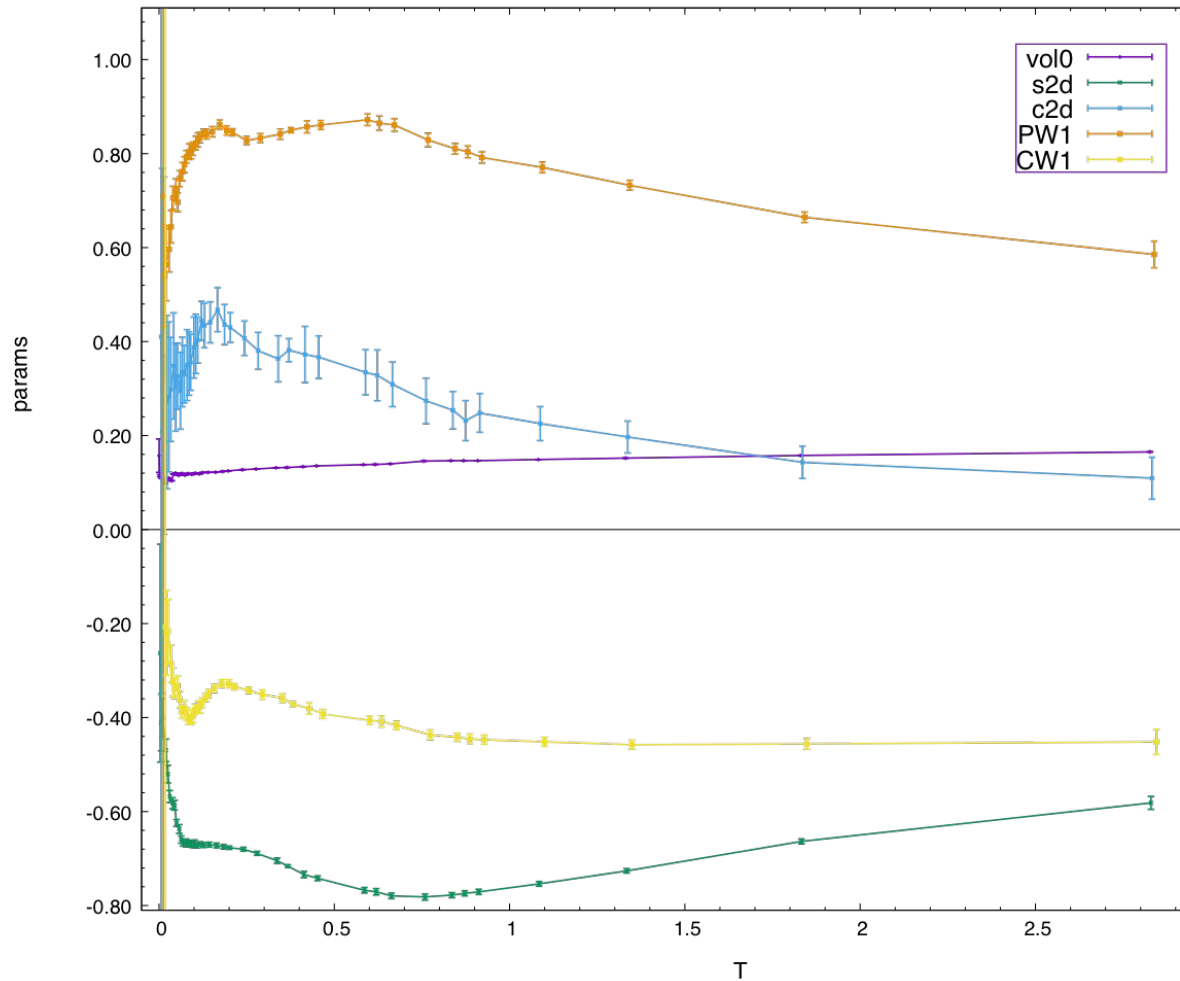
**For reference: 3w before
big covid crash...**

SPX 20200218 15:00

C15PM Param Term-Structure

First 3 params...

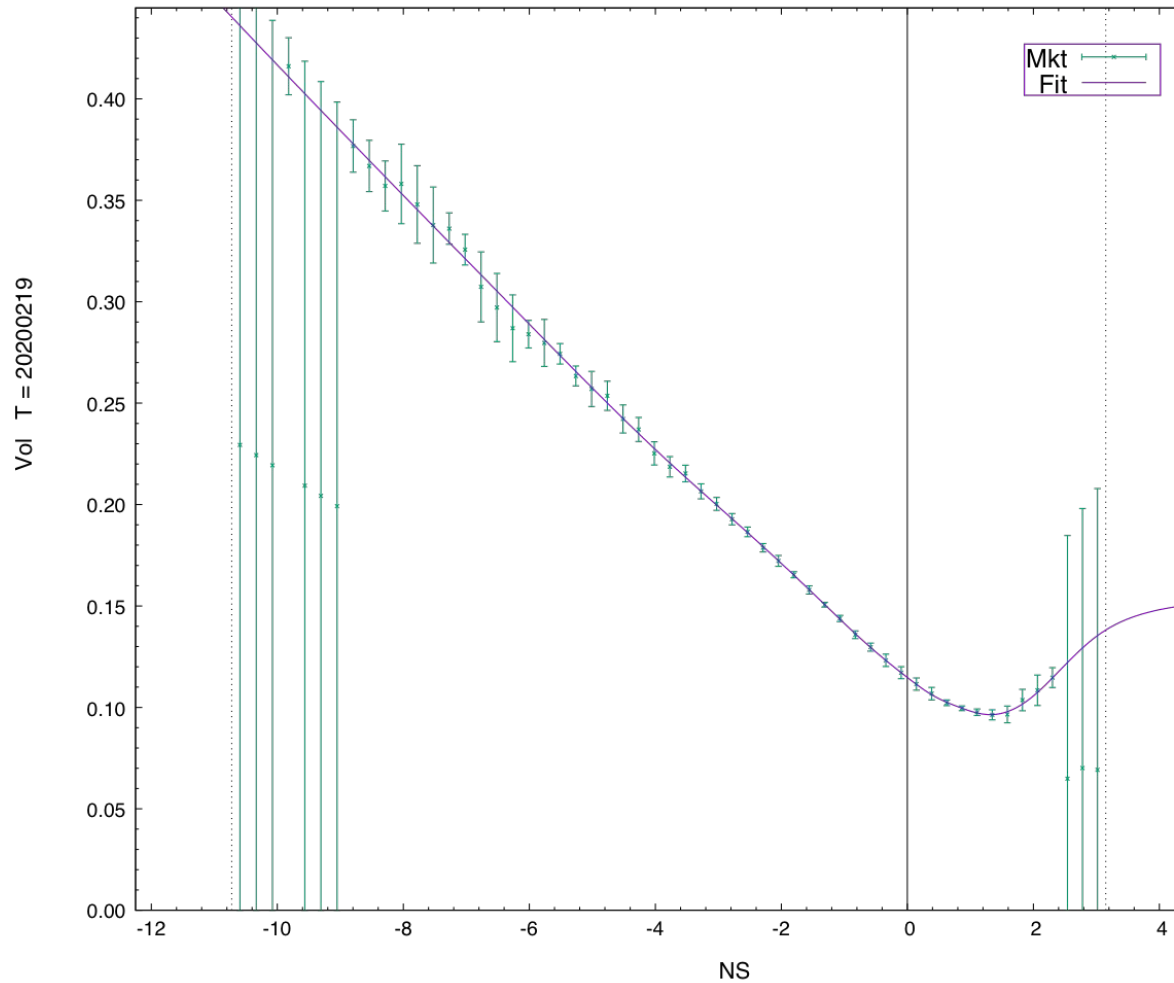
$s2(T)$ a bit unusual...



SPX 20200218 15:00

C15PM Param Term-Structure

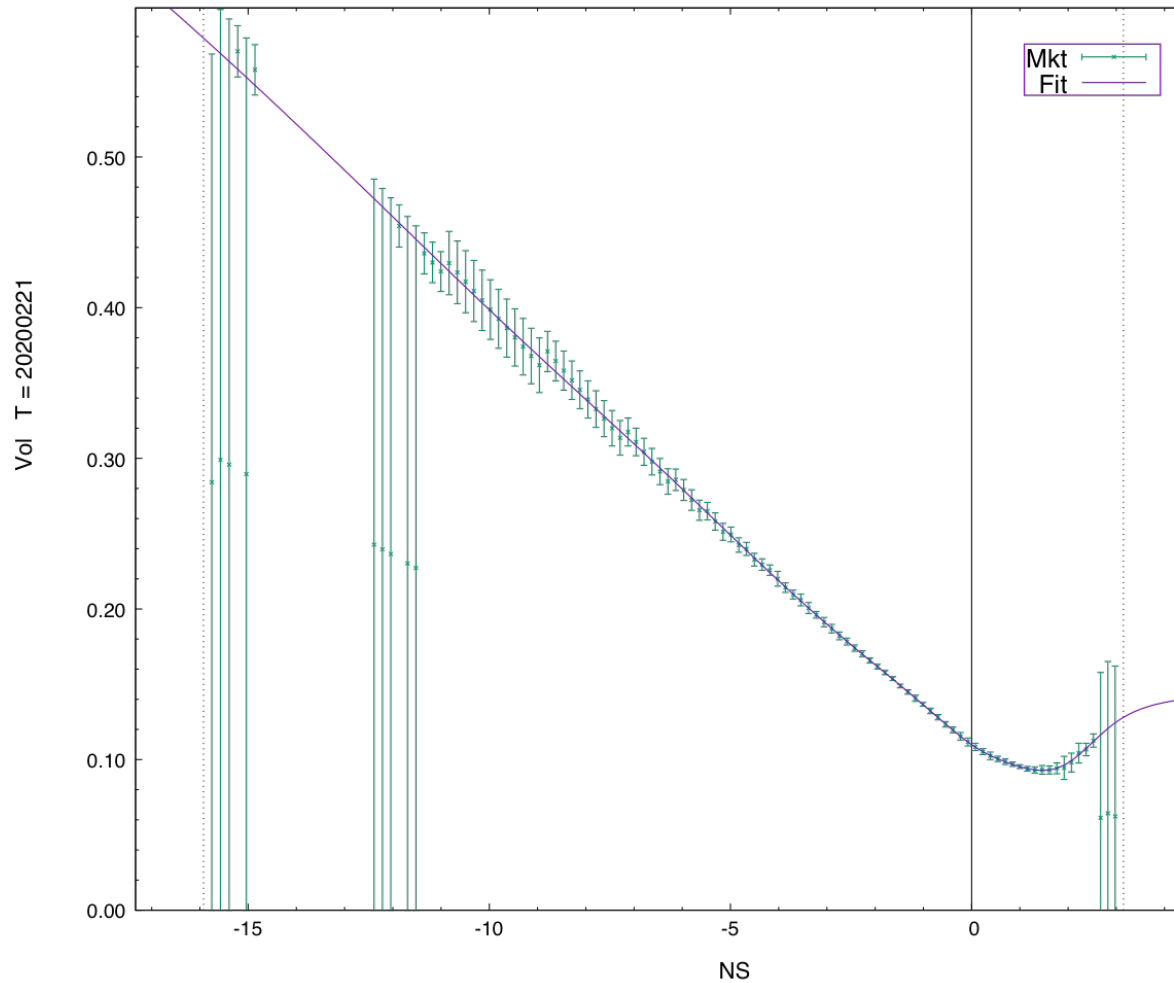
First 5 params... meaning?



SPX 20200218 15:00

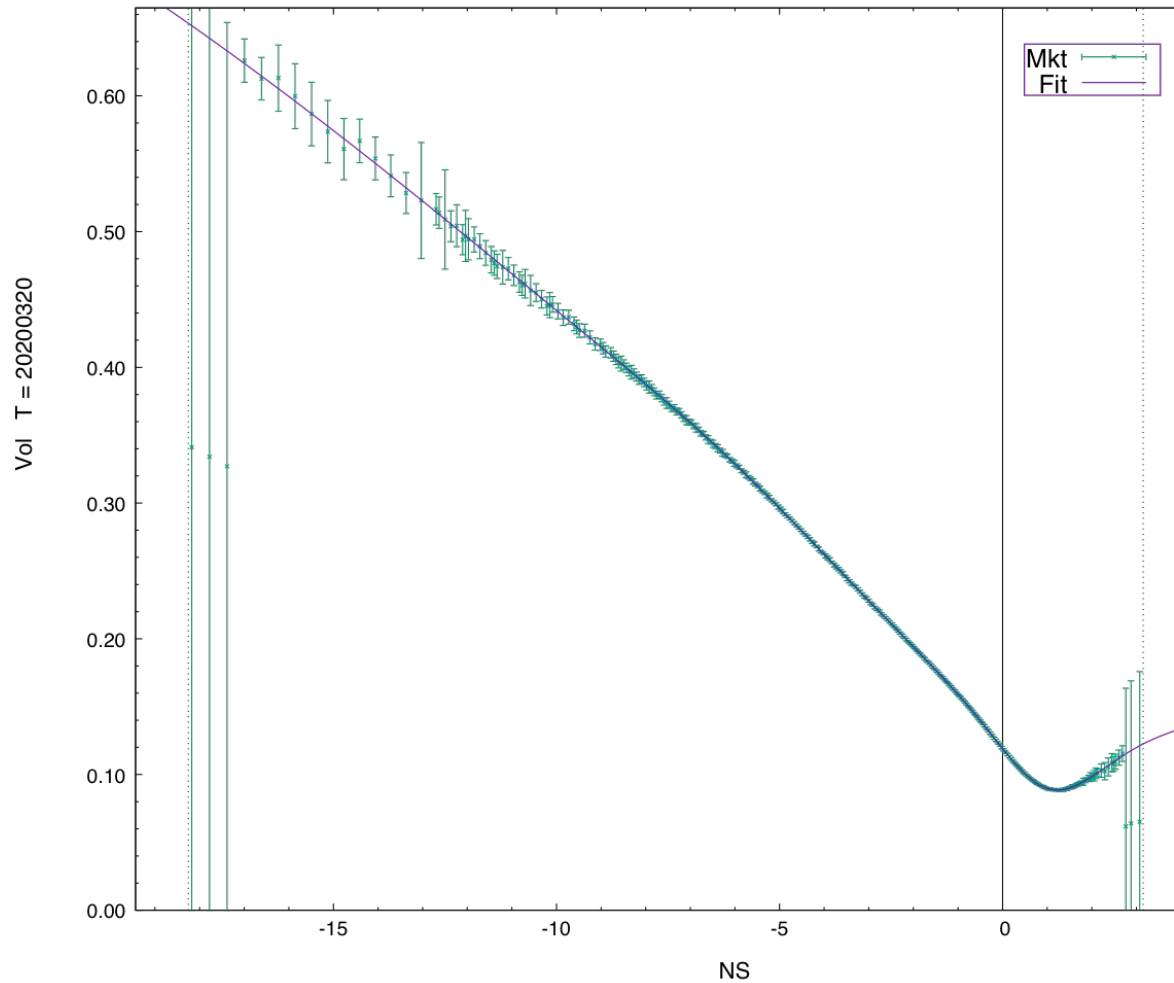
C15PM T = 1d

$$z := NS := \frac{\ln(K/F)}{\sigma_0 \sqrt{T}}$$



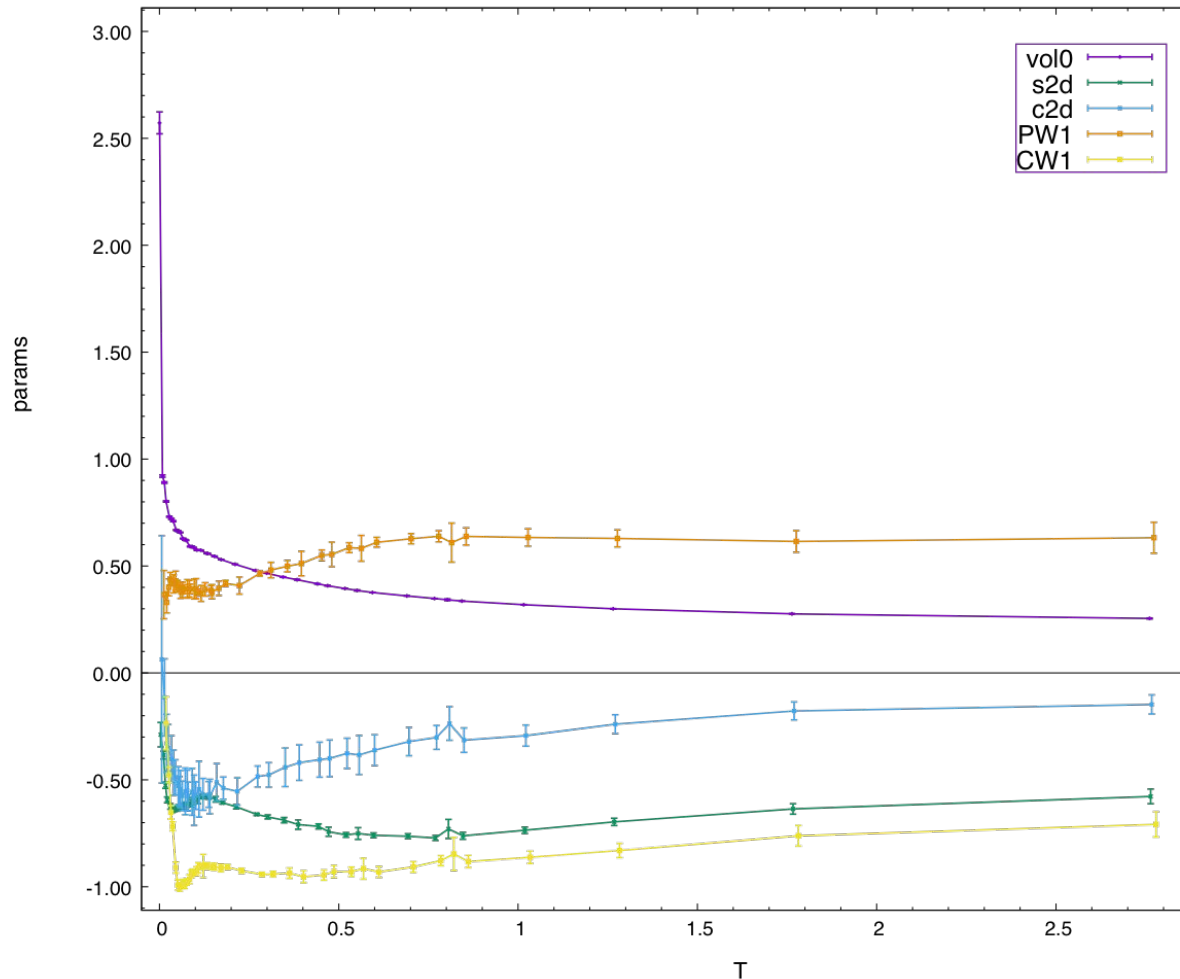
SPX 20200218 15:00

C15PM T = 3d



SPX 20200218 15:00

C15PM T = 1m



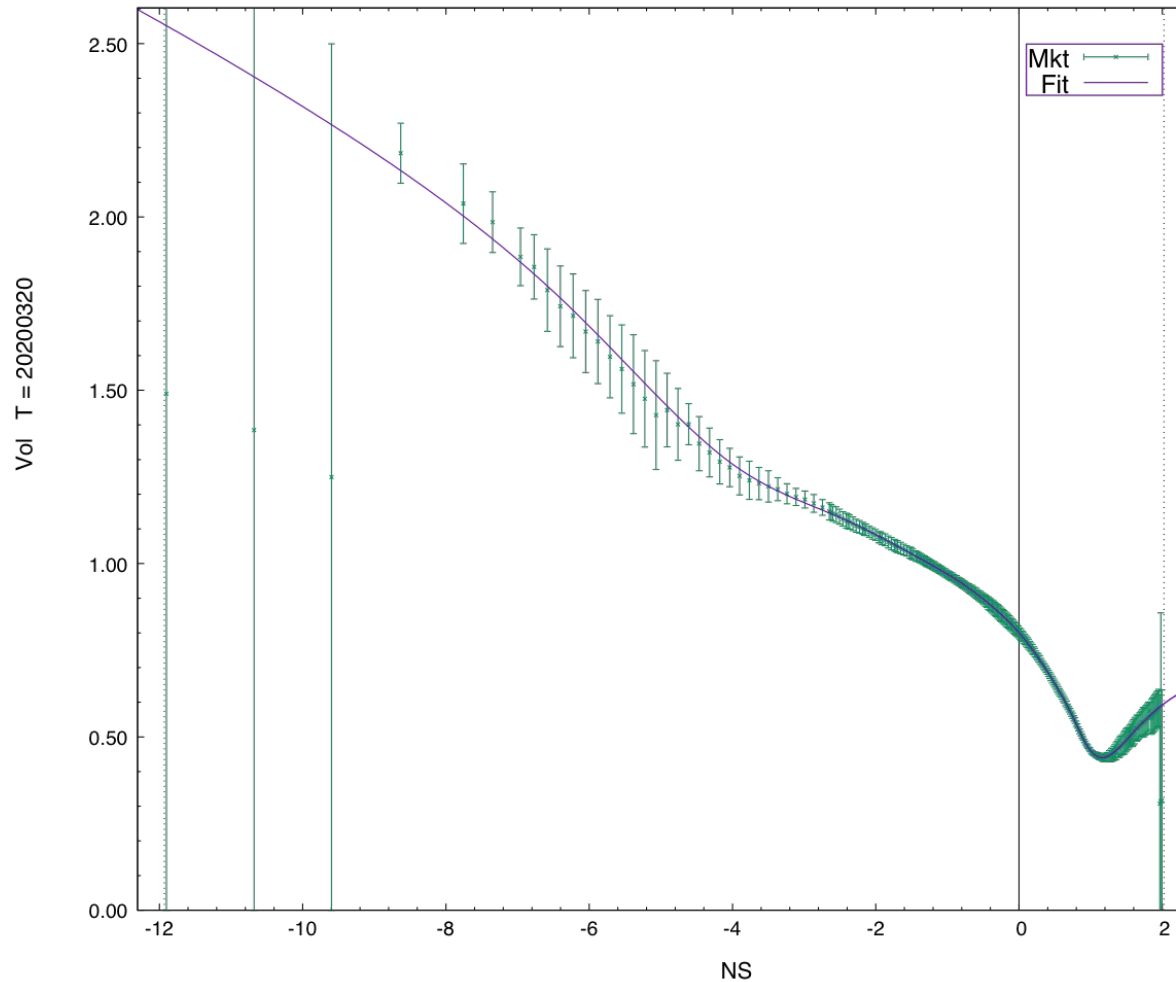
SPX 20200313 15:00

C15K Param Term-Structure during the **covid crash**

First 5 params...

All **c2** < 0 !!

Super-steep near call wing: **CW1**

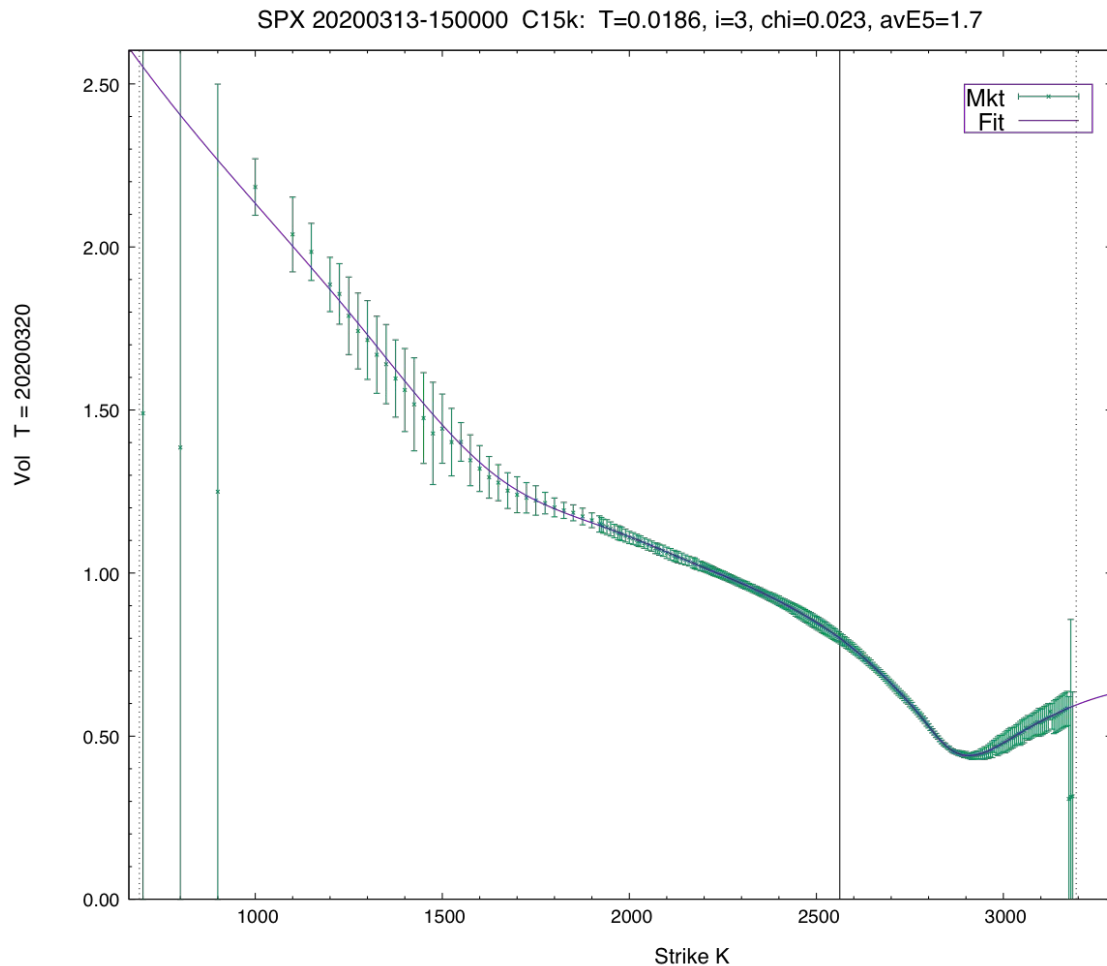


SPX 20200313 15:00

C15K T = 1w, in NS-space

Very compressed CW.

If fit followed PW more closely there would be fly arb...



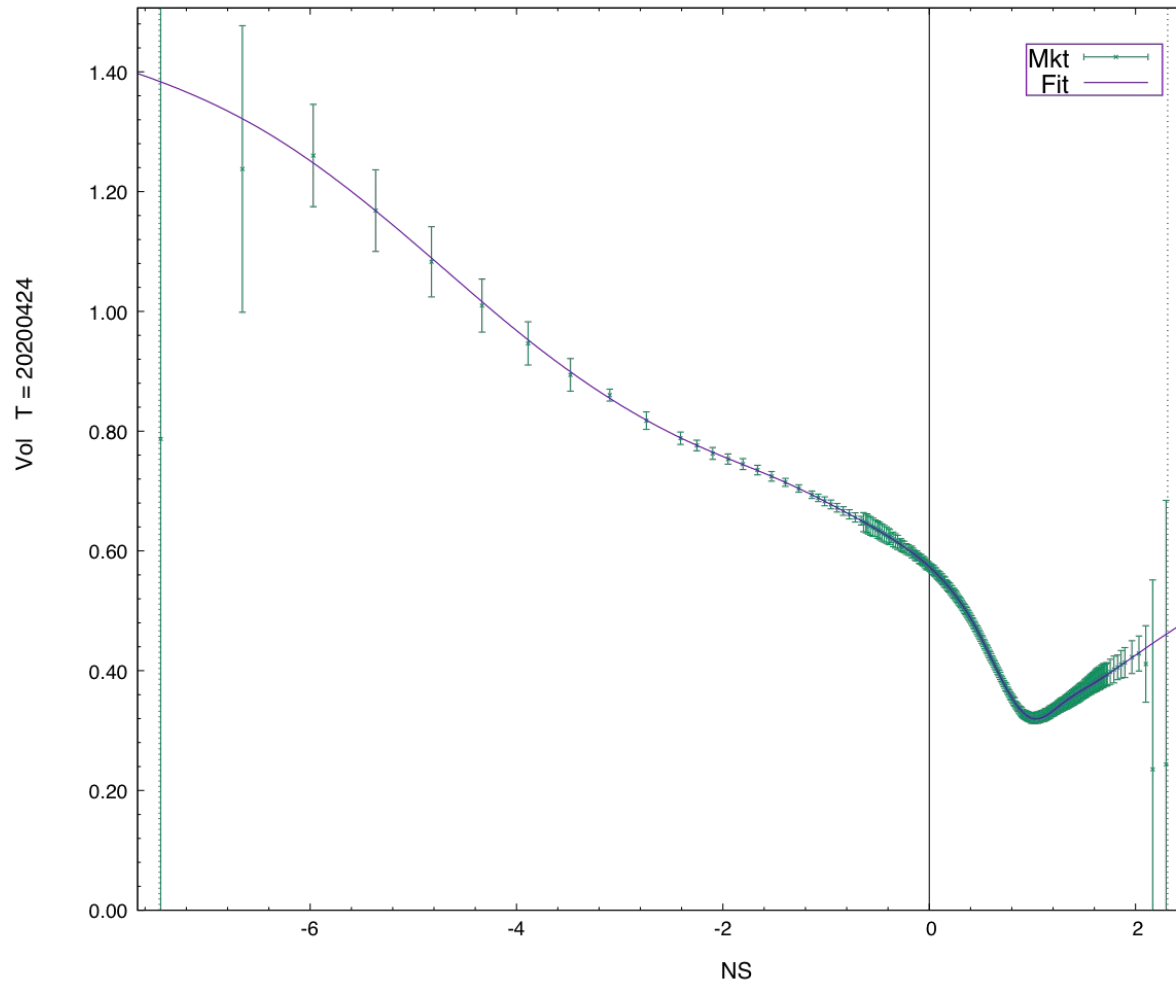
SPX 20200313 15:00

C15K $T = 1w$, in K-space

Very compressed CW.

If fit followed PW more closely there would be fly arb...

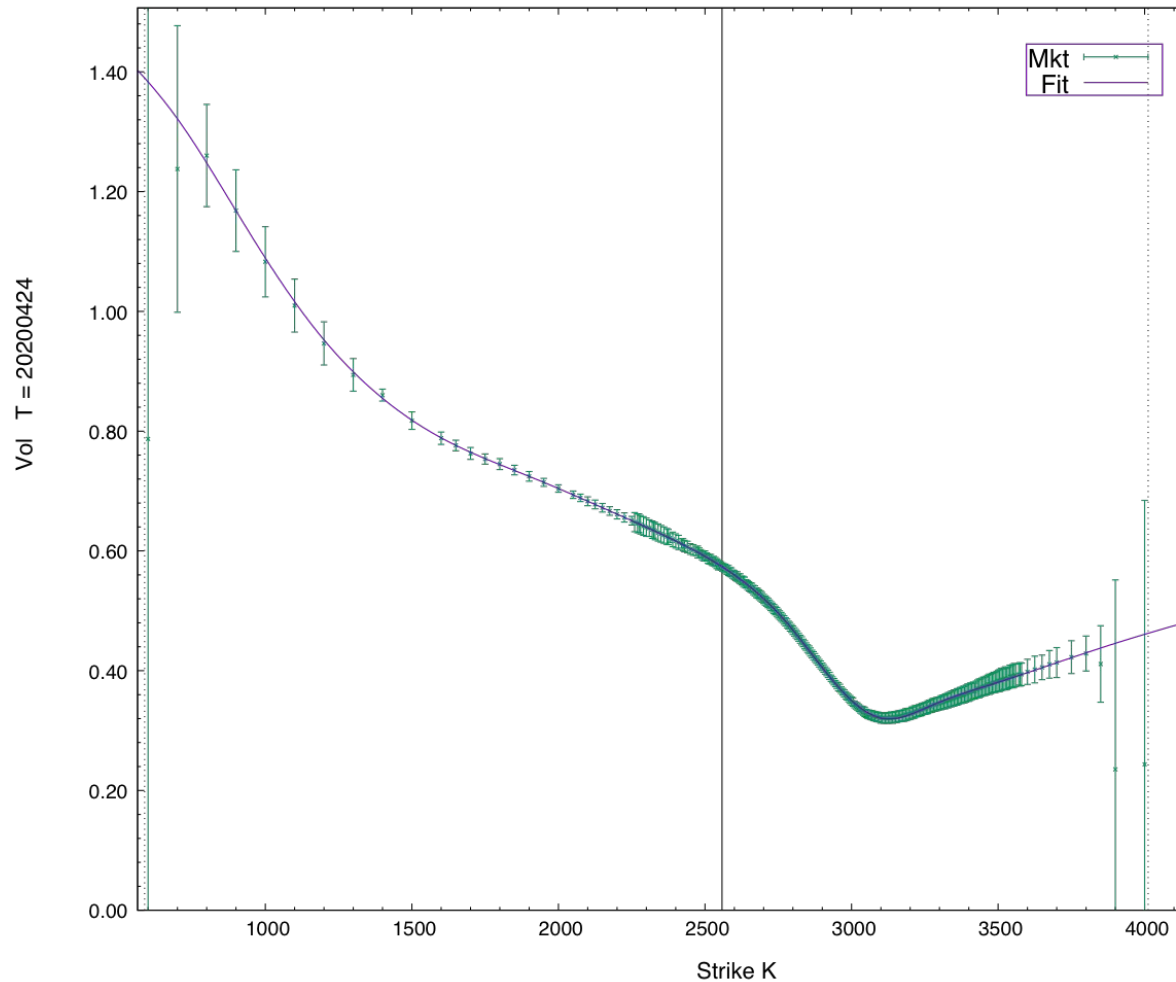
(Pretty well-functioning market over $nK=379$ strikes here...)



SPX 20200313 15:00

C15K T = 6w, in NS-space

Very compressed CW, very sharp knee...



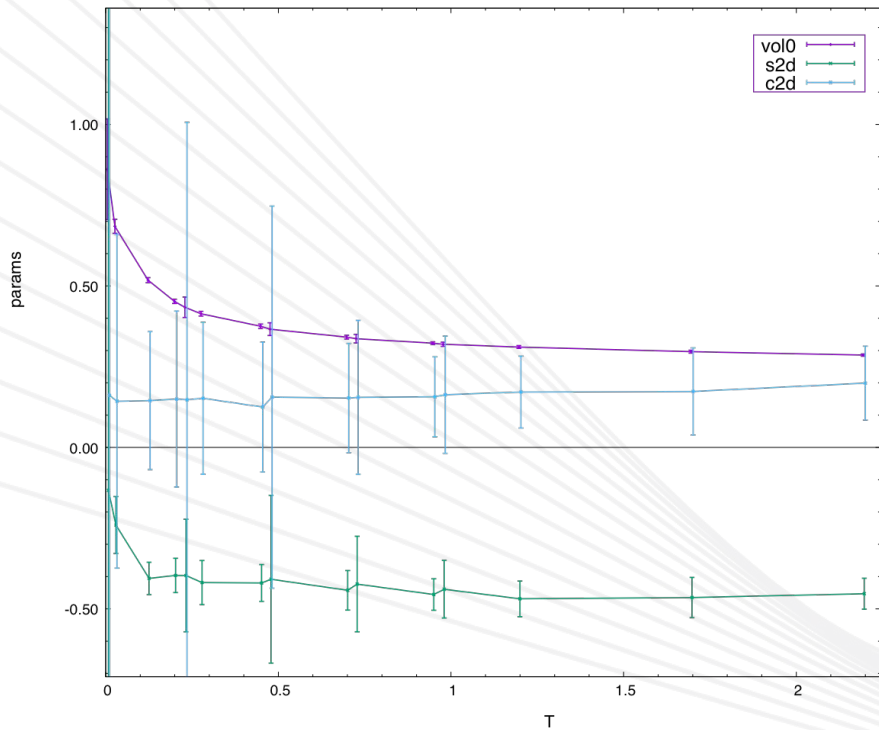
SPX 20200313 15:00

C15K T = 6w, in K-space

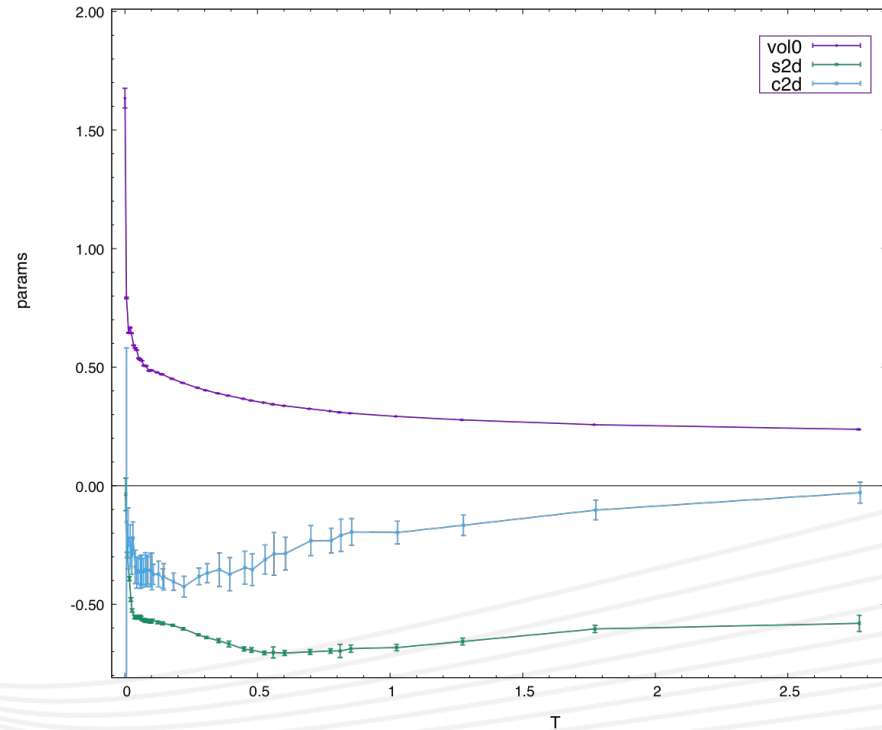
Very compressed CW, very sharp knee...

Parameter TS: 2008 versus 2020

Parameter TS SPX 20081008-160000 C8, $\chi_{\text{Av}}=0.028$

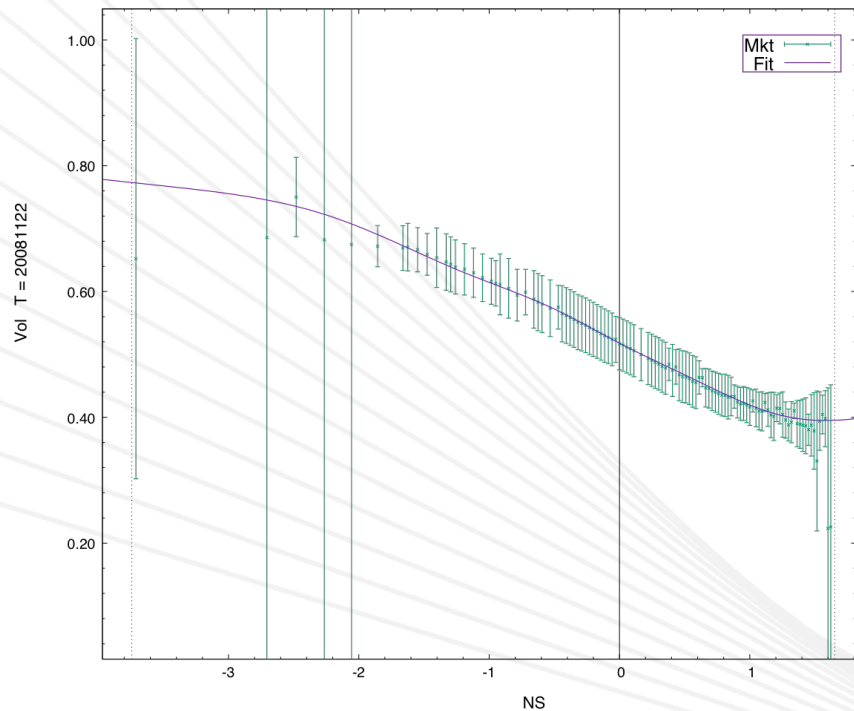


Parameter TS SPX 20200311-150000 C15k, $\chi_{\text{Av}}=0.014$, $F_0=2742.65$

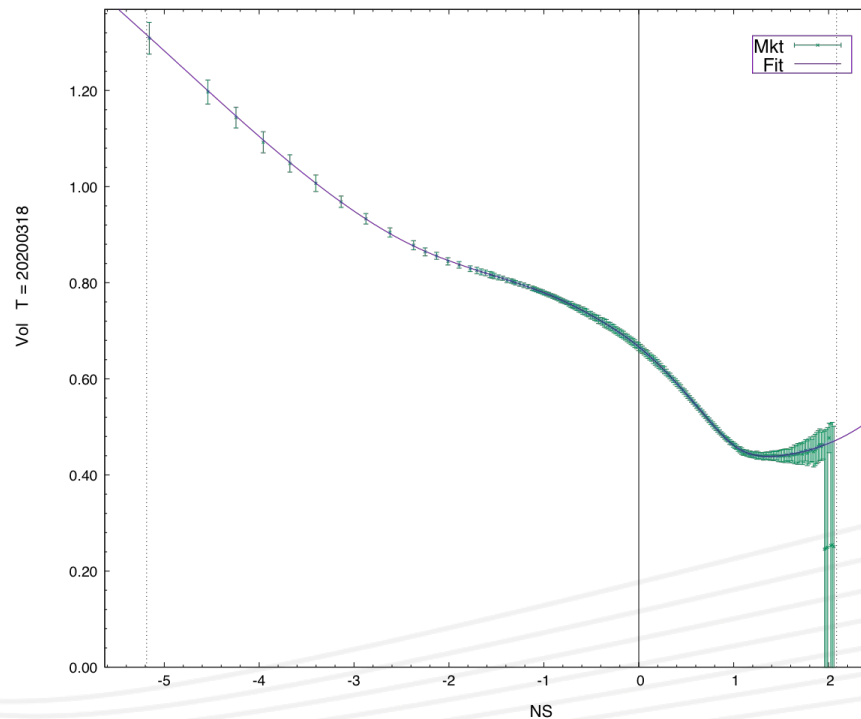


Vol Skews: 2008 versus 2020

SPX 20081008-160000 C8: $T=0.1227$, $i=2$, $\chi=0.027$, $avE5=8.3$



SPX 20200311-150000 C15k: $T=0.0193$, $i=3$, $\chi=0.019$, $avE5=0.7$



Vol Curve Statistics

- There are ~5630 names in OPRA (Oct 2023). We find, roughly, for bias-free fits:
 - 4100 (73%) can be fit with **S3**/SSVI. (S5/SVI: 70 or 1.2%)
 - 650 (11.5%) can be fit with **C5**.
 - 700 (12.4%) can be fit with C6, **C7***, C8* C9*.
 - There are a 70 inverse curves (C6C+) for VIX, VXX, (inverse) leveraged ETFs, low-priced stocks.
 - The remaining 50 (0.9%) names require higher **C10 – C16** curves – the most liquid names!
- SPX/SPY/ES require ~16-18 parameters (for some terms) to get bias-free fits of all options down to zero-bids. Some OMMs use ~25 params for SPX.
- Big tech names and (other) global indices require 9 –15 params per term.
- There has been a relentless drive towards higher curves, to fit tighter spreads and wider (normalized) strikes ranges.
 - Empirically, roughly (for OPRA universe): $nParams \approx (nOptions / 5)^{1/3}$

Event Modeling

- Modeling an event like earnings properly requires a jump model.
 - With events at specific, non-random time(s).
- Minimal proper model: Two “**Merton Event Jumps**”, on top of a diffusive process, described by a “clean” aka “background” vol surface.
 - Various assumptions are possible about how to combine the clean surface with jumps.
 - Pricing: sum of Black formulas with an integral over jump sizes.
 - This is a ~proper model, unlike just describing the pdf of some expiry with a sum of log-normals, aka the Log-Normal Mixture “Model”.
 - So, one can ask if different expiries are **consistent with the same jumps**, and similar non-trivial questions.

AMZN 2023-08-02 earnings

Calibrating three 2-jump models, with 2,3,4 parameters:

ASYMMETRIC_DISCRETE_JUMP

$J = [0.955, 1.071]$

$p = [0.613, 0.387]$

$s = [0.000, 0.000]$

ASYMMETRIC_MERTON_JUMP

$J = [0.956, 1.084]$

$p = [0.654, 0.346]$

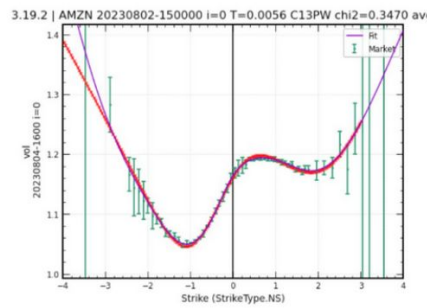
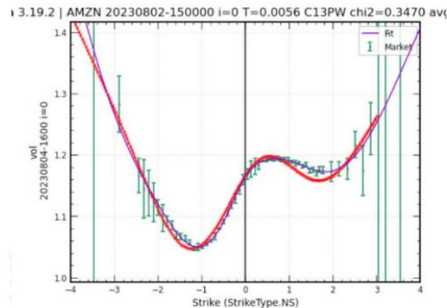
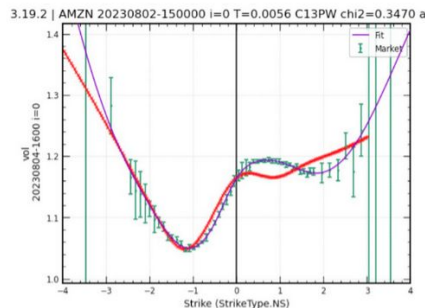
$s = [0.045, 0.045]$

ASYMMETRIC_MERTON_JUMPS

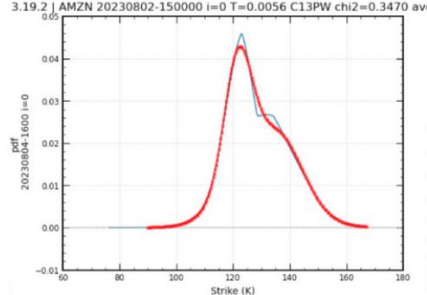
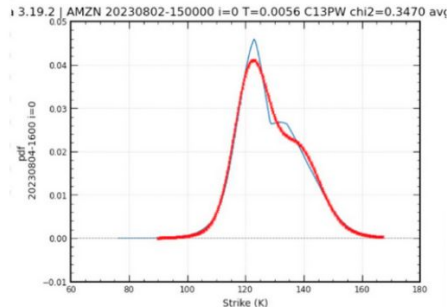
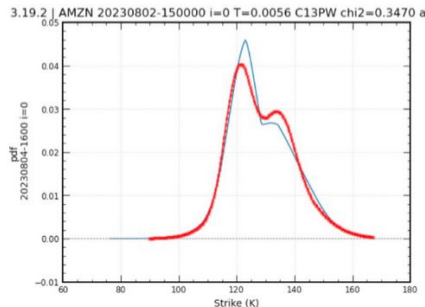
$J = [0.944, 1.059]$

$p = [0.516, 0.484]$

$s = [0.037, 0.059]$



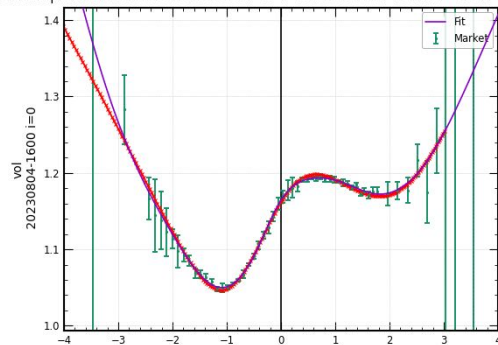
vctClean = C5
nAfter = 3



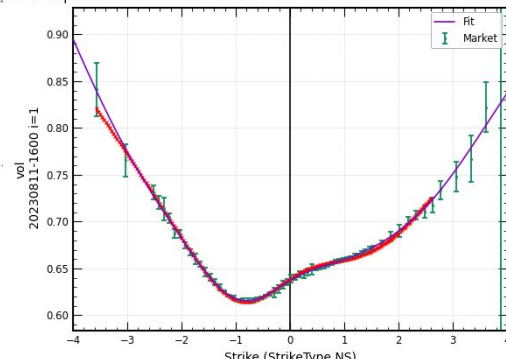
AMZN 2023-08-02 earnings

The best 2-jump model works for three expiries after earnings:

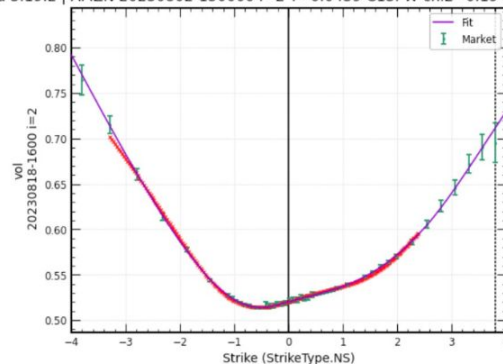
3.20.1 | AMZN 20230802-150000 i=0 T=0.0056 C13PW chi2=0.3470 avgE5



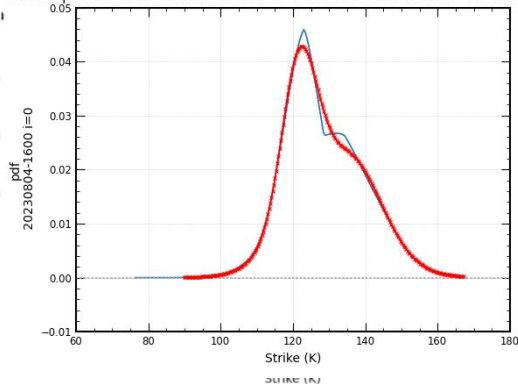
3.20.1 | AMZN 20230802-150000 i=1 T=0.0248 C13PW chi2=0.1796 avgE5



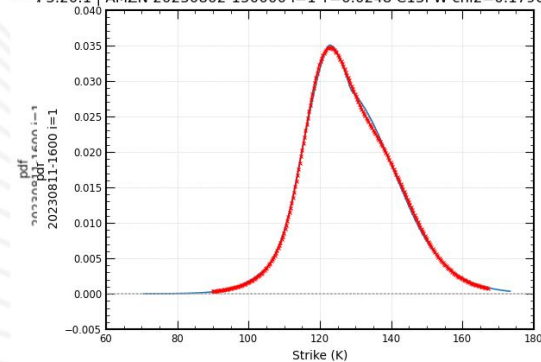
3.19.2 | AMZN 20230802-150000 i=2 T=0.0439 C13PW chi2=0.1948 avgE5



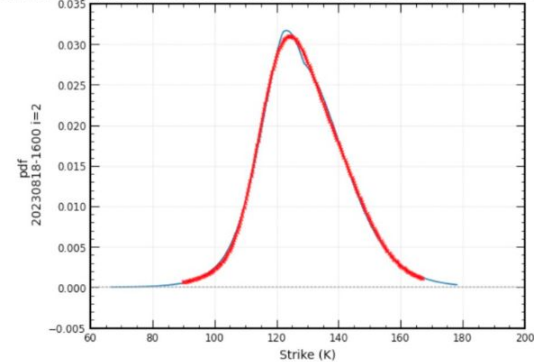
3.20.1 | AMZN 20230802-150000 i=0 T=0.0056 C13PW chi2=0.3470 avgE5



3.20.1 | AMZN 20230802-150000 i=1 T=0.0248 C13PW chi2=0.1796 avgE5



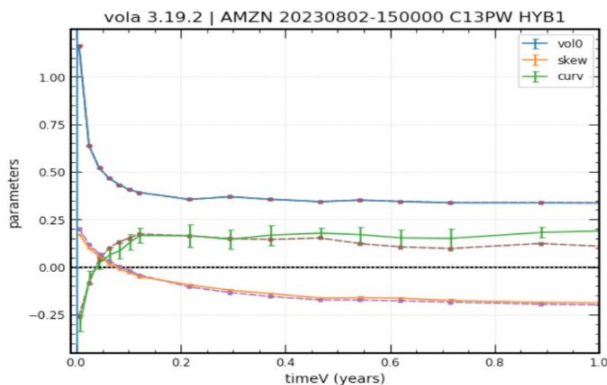
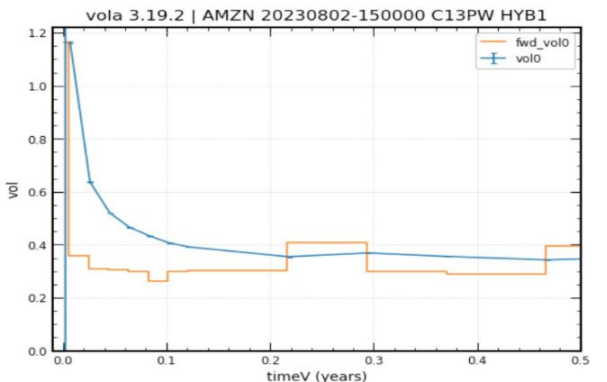
3.19.2 | AMZN 20230802-150000 i=2 T=0.0439 C13PW chi2=0.1948 avgE5



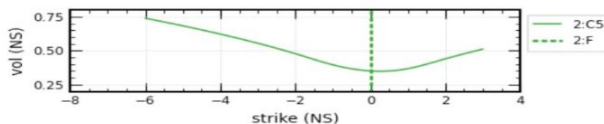
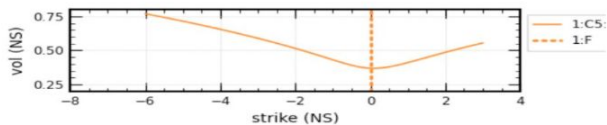
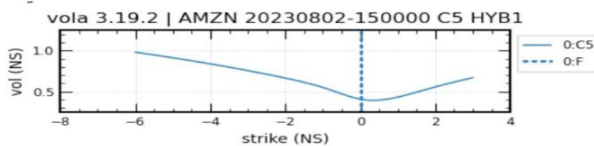
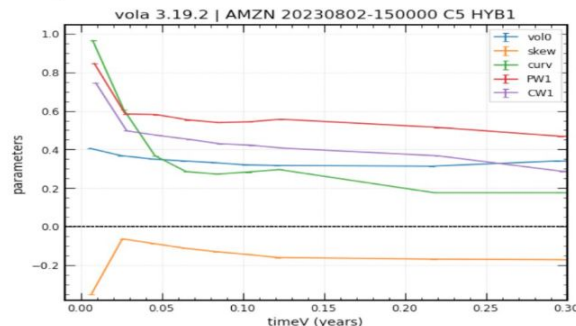
AMZN 2023-08-02 earnings

Though the clean VS is not quite as clean as we would like:

Dirty



Clean

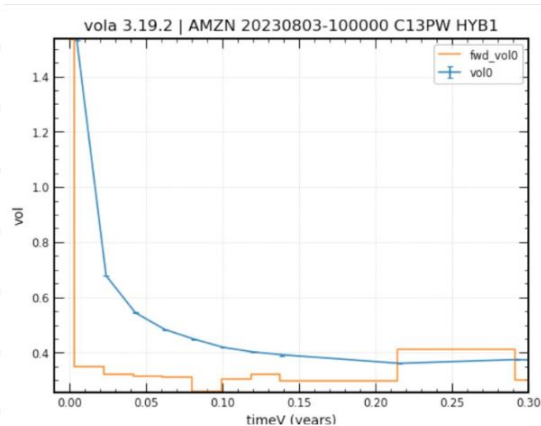


nAfter=3
vctC = C5
MertonJumps

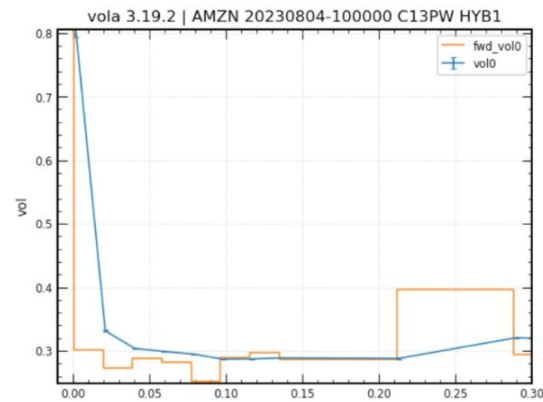
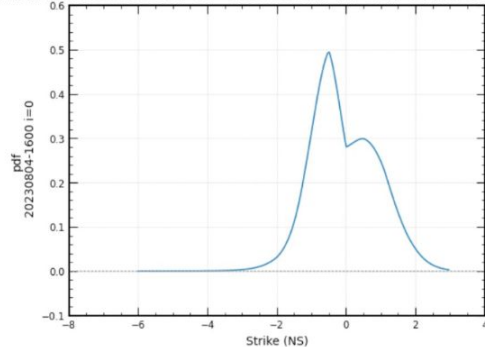
J= [1.0594025258161424, 0.9441995069221296]
p= [0.48436658703542335, 0.5156334129645767]
s= [0.0588065555709617, 0.03688014214112789]

AMZN 2023-08-02 earnings

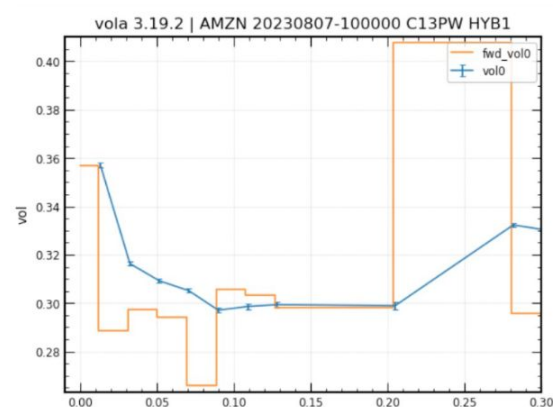
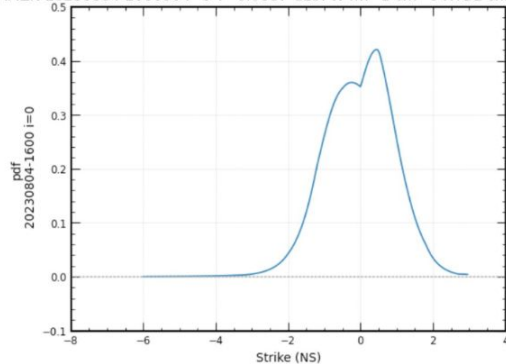
The event is not quite discrete in time, there are “**aftershocks**”:



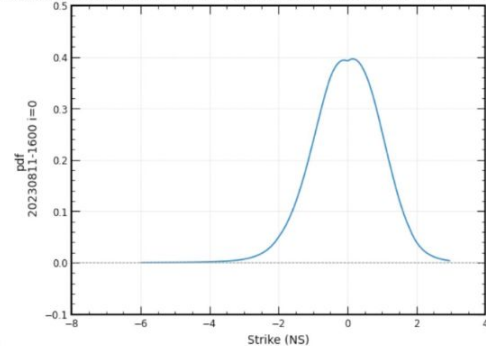
AMZN 20230803-100000 i=0 T=0.0034 C13PW fm=1 cm=0 HYB1 chi2=



AMZN 20230804-100000 i=0 T=0.0007 C13PW fm=1 cm=0 HYB1 chi2=



AMZN 20230807-100000 i=0 T=0.0116 C13PW fm=1 cm=0 HYB1 chi2=



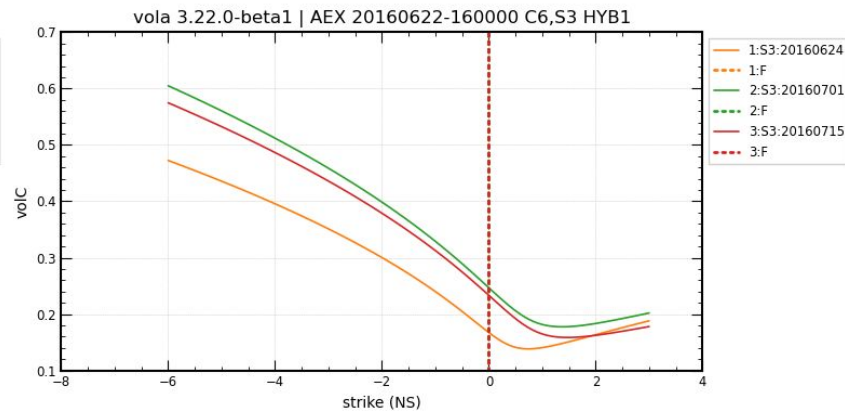
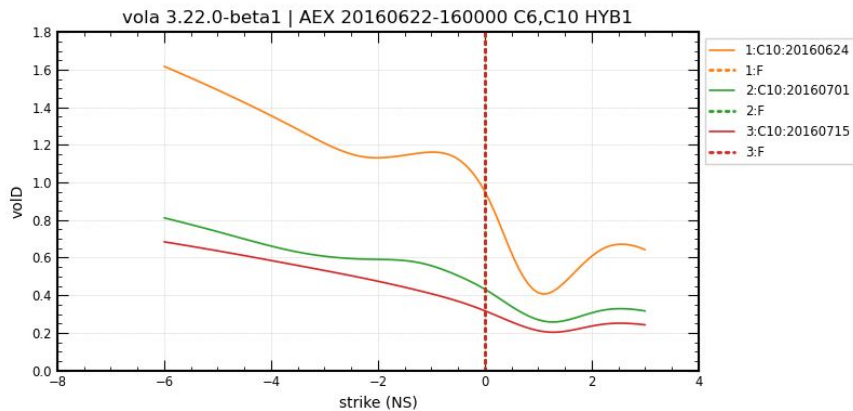
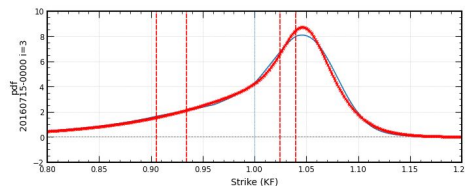
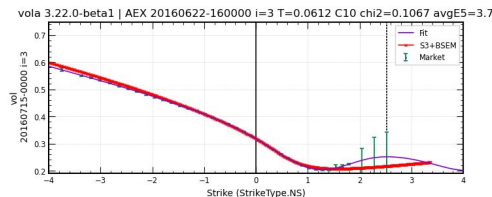
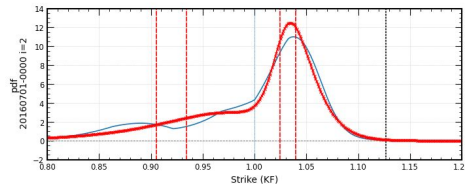
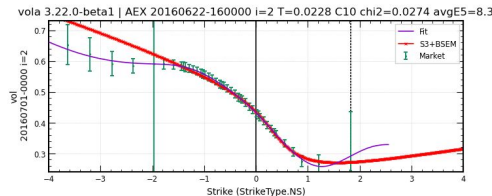
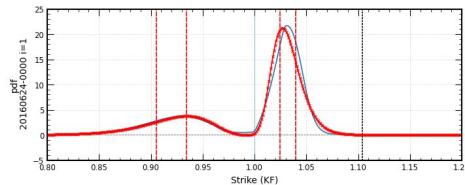
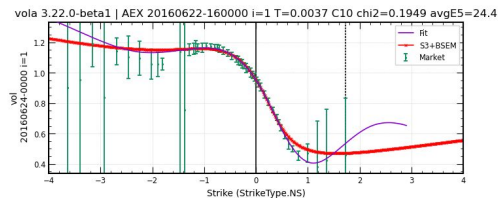
AEX before Brexit

Calibrated jump params:

$$J = [0.9201, 1.0322]$$

$$p = [0.2870, 0.7130]$$

$$s = [0.0146, 0.0076]$$



SPX before 2024 election

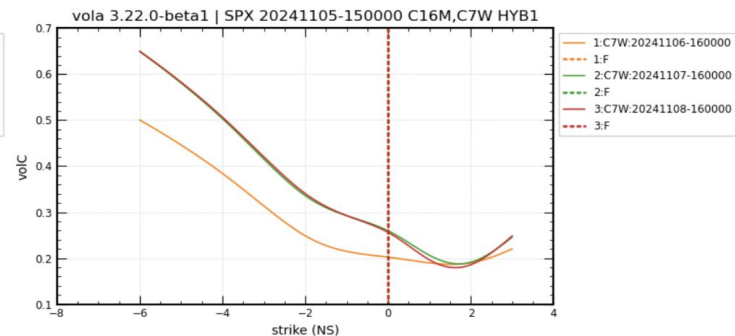
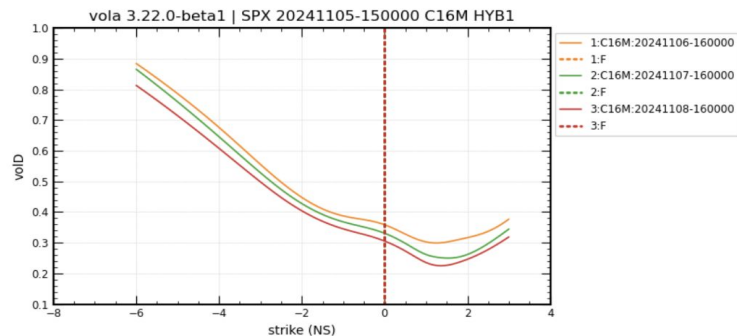
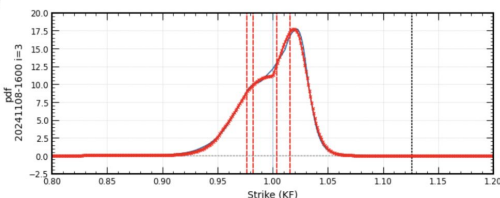
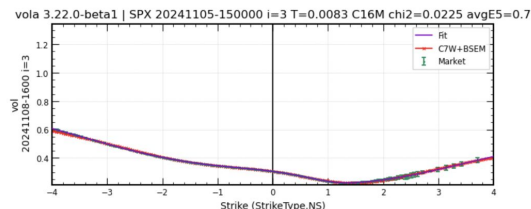
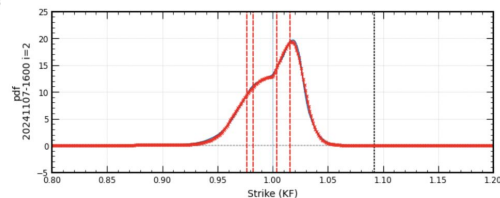
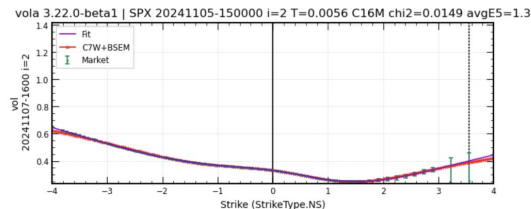
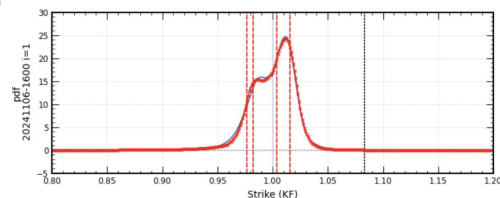
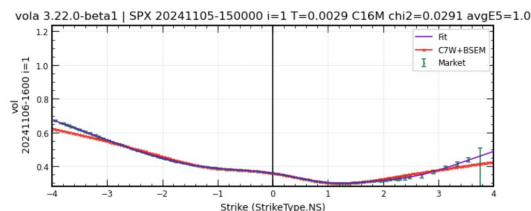
Calibrated jump params:

$$J = [0.9795, 1.0096]$$

$$p = [0.3183, 0.6817]$$

$$s = [0.0028, 0.0061]$$

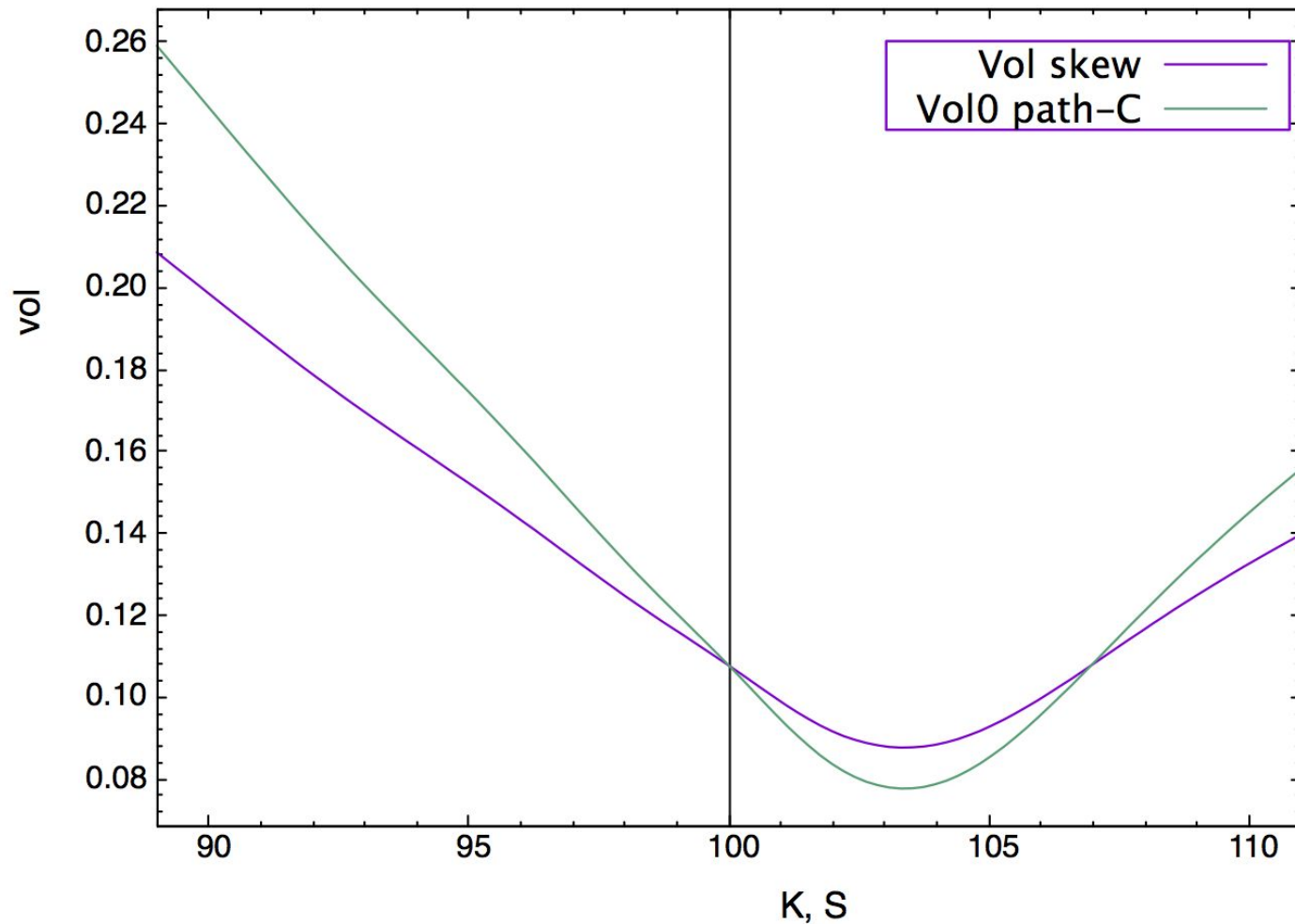
- Calibration performed with just the $i=1$ expiry, followed by dirty \rightarrow clean Vol surface conversion performed with the calibrated event shown above.
- The calibrated clean Vol Surface exhibits the presence of an aftershock.



Spot-Vol Dynamics: Basics

- **Shape** (by NS or Δ) is much more **stable** than overall vol level (vol0 aka ATF vol).
- **ATF vol dynamics** is very well described by one dimensionless number, **SSR** aka vol sensitivity aka super-skew, which is the ratio of vol0-path & skew slopes.
- Very **simple dynamics** in terms of NS **vol parameters** (e.g. just ATF vol), gives **complicated vol-by-strike dynamics**, that actually describes market moves.
 - It also gives the correct adjusted (aka smart aka **skew**) **delta** and gamma (see LinkedIn article).
- We will illustrate each of these points.

ATF Vol path (C8, volSensi = 1.5, clampFac = 0.2)



Spot-Vol Dynamics

ATF “vol path”

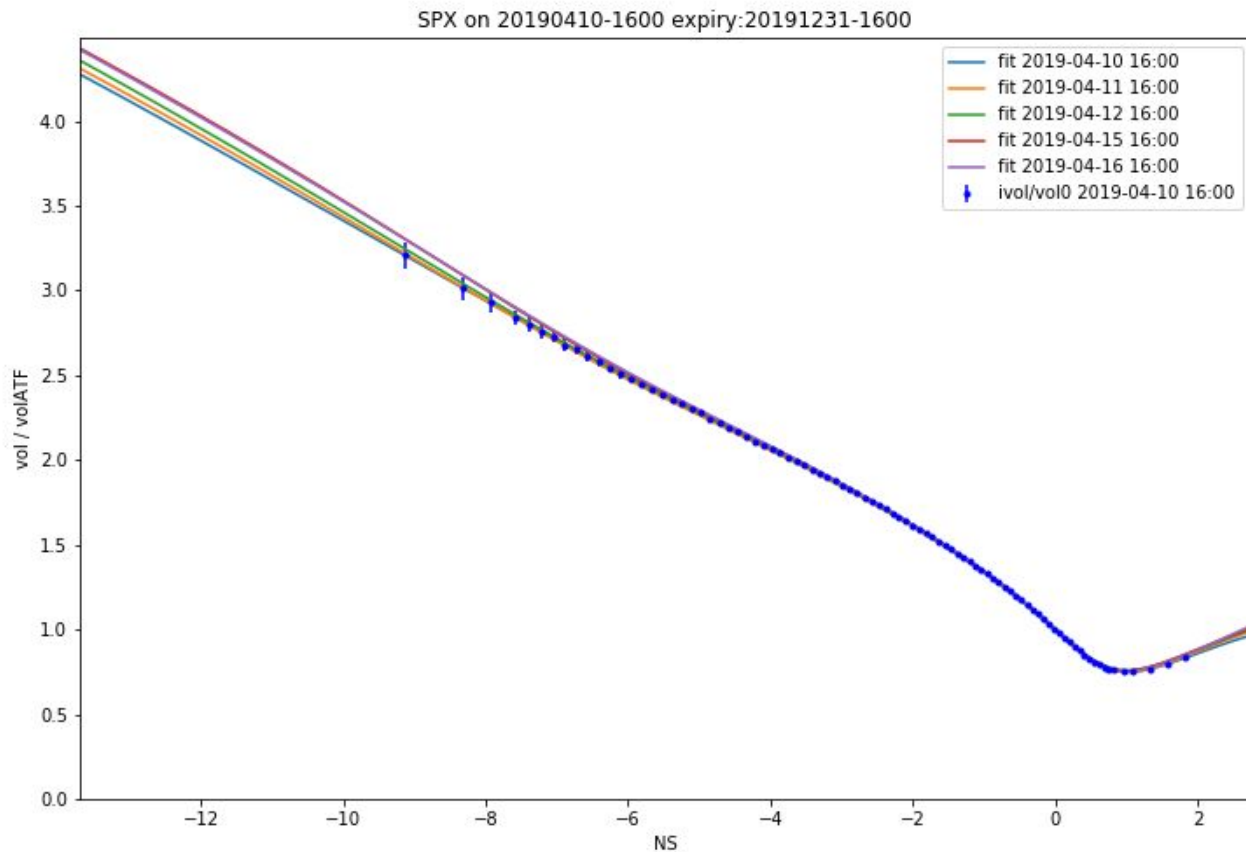
SSR = 1.5

“Along curve”

No clamps

SPX Spot-Vol Dynamics: Then and Now

- In the olden days:
 - Virtually no shape dynamics.
 - Overall vol level dynamics described very well by one SSR with little term-structure (TS).
 - $1 < SSR < 2$, with 2 reached only on big down days. Typical value $SSR=1.3$.
- Nowadays:
 - There is often **term-structure** in SSR, with $SSR(T > 1y)$ closer to typical values.
 - There is occasionally, e.g. on some big down days, **shape dynamics**, eg in c2.
 - **$SSR > 2$ and $SSR < 1$** can happen, on short end.
 - Some horizon dependence (1min, 5min, etc), including intraday vs overnight differences.
 - More “fluctuations”, in **path-dependent** manner (cf. Guyon), around typical values.
 - Open Q: How strong is path-dependency effect relative to levels set by “SSR regime” ?

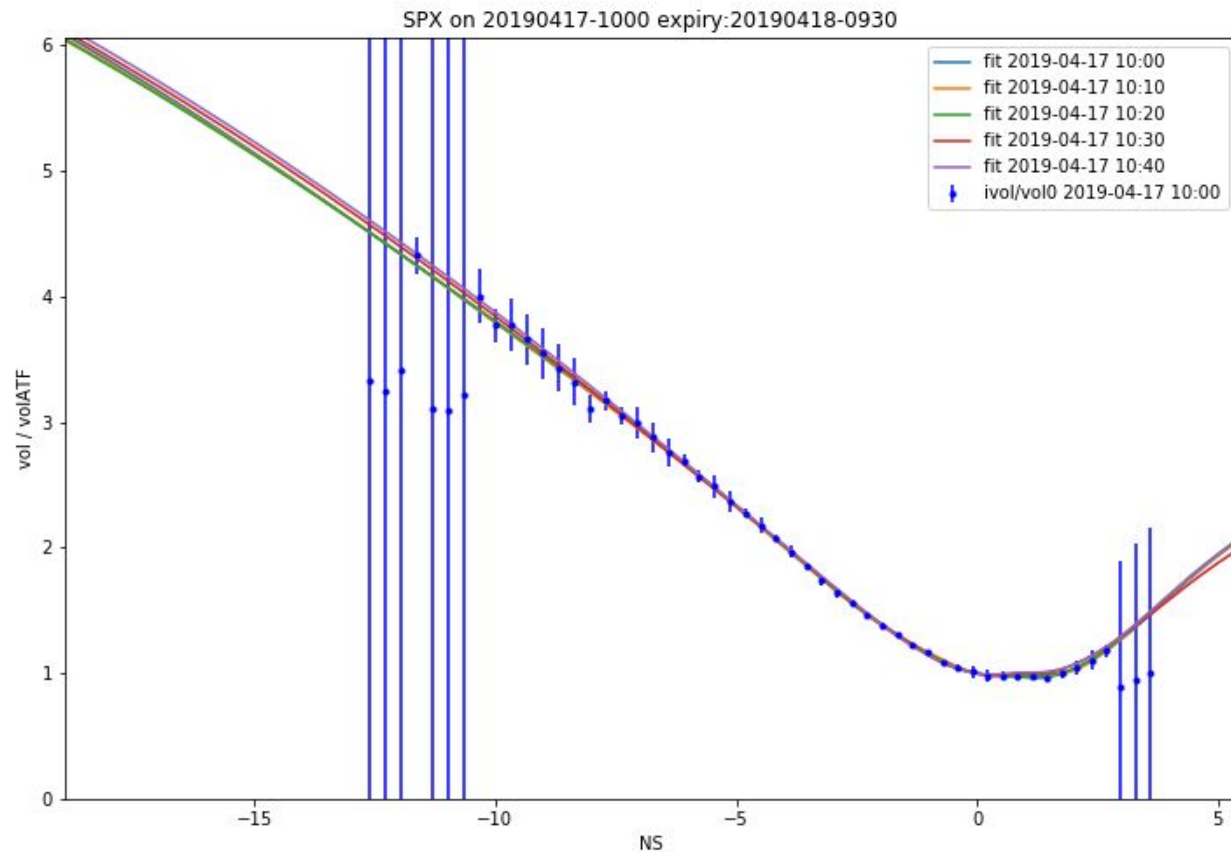


Stability of NS Shape

SPX 20190410 T = 9m

Shape **stable over many days**,
while underlier moves around.

Also, no floppy wings!

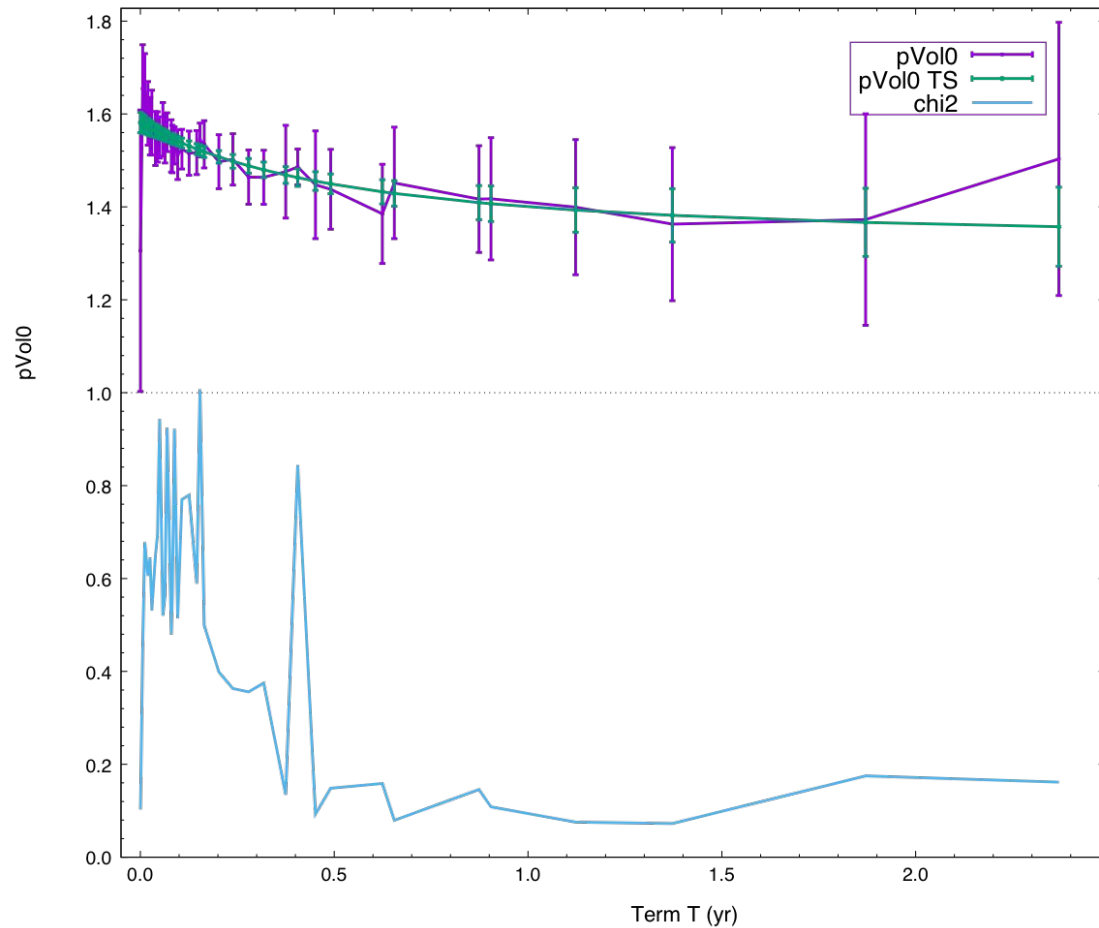


Stability of NS Shape

SPX 20190410 T = 1d

Shape stable even on last day

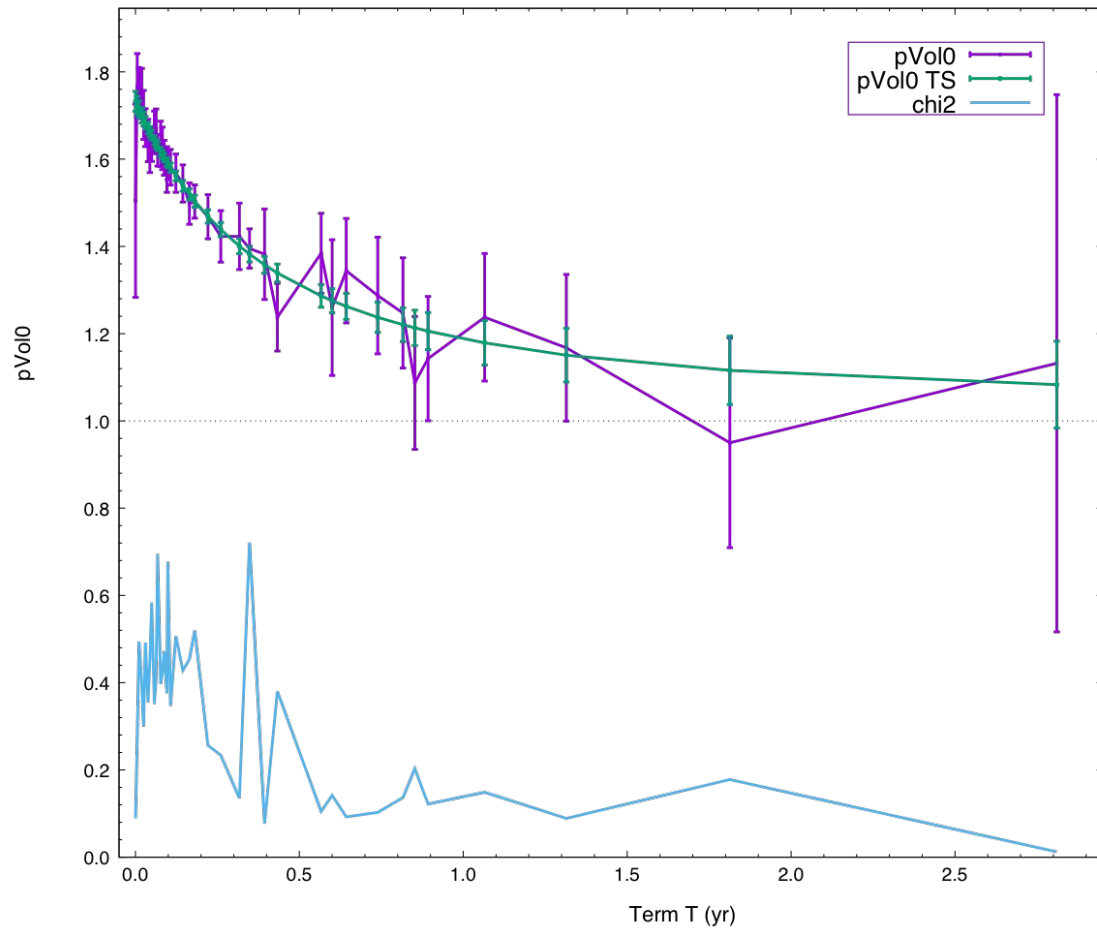
Also, no floppy wings!



SPX 20190805

Vol sensitivity (SSR) term-structure

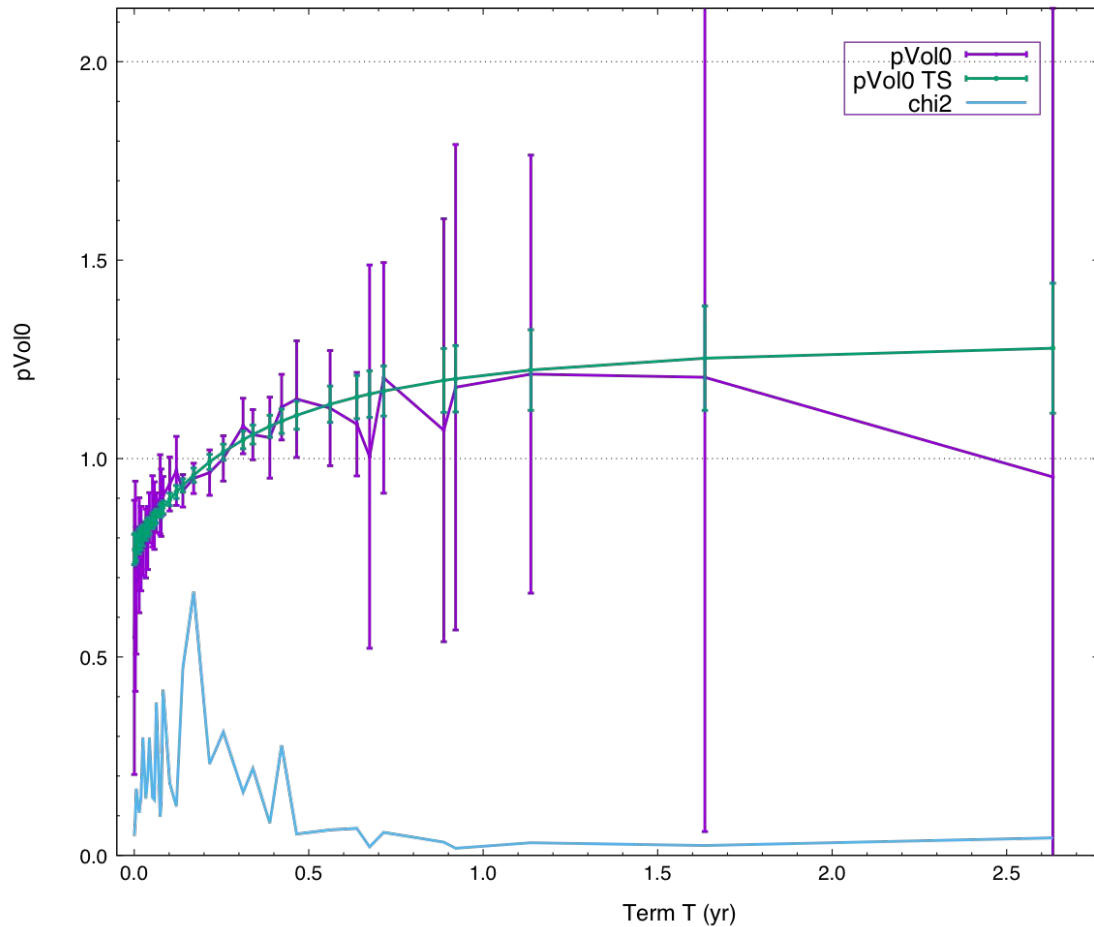
Parametric fit for robustness on small data sets (can be done intra-day)



SPX 20200224

Vol sensitivity (SSR) term-structure

Parametric fit for robustness on
small data sets



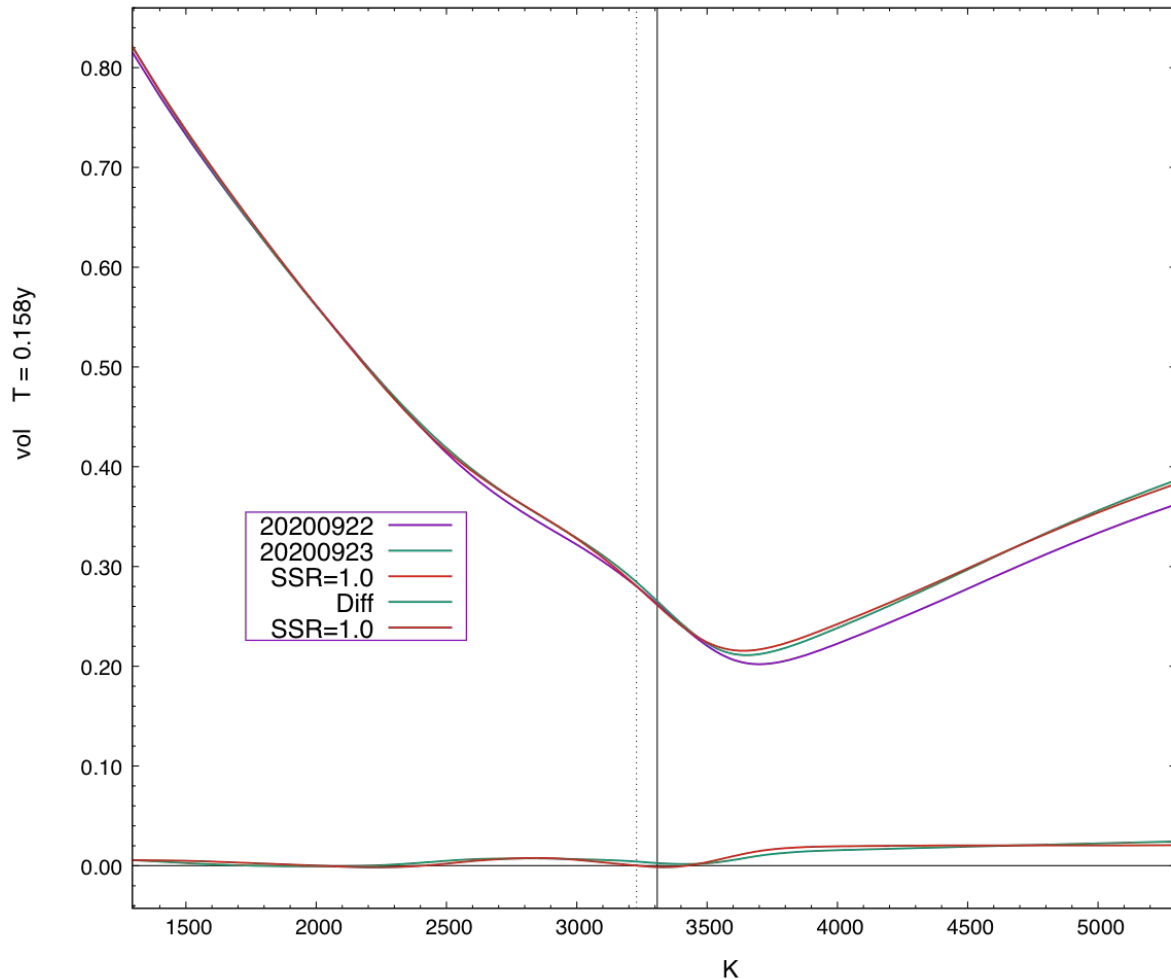
SPX 20200429

Vol sensitivity (SSR) term-structure

On up-days can be upward-sloping,
and SSR < 1 at least for some terms

Spot-Vol Dynamics Myths

- **Myth: Common “vol regimes” are “sticky-by-strike” or “sticky-by-delta”**
 - Sticky-strike scenarios are still commonly used by risk departments.
- In equities, at least, **neither has happened for 20y+.**
- Sticky-delta implies $SSR=0$ for all terms. Never happens.
- Sticky-strike implies $SSR=1$ (i.e. sticky-strike around ATM).
 - Even when vols are sticky-by-strike around ATM, they never are in the wings.
 - There are many examples. Let's look at some.



Close-to-close spot vol dynamics

SPX 2020-09-22 to 2020-09-23

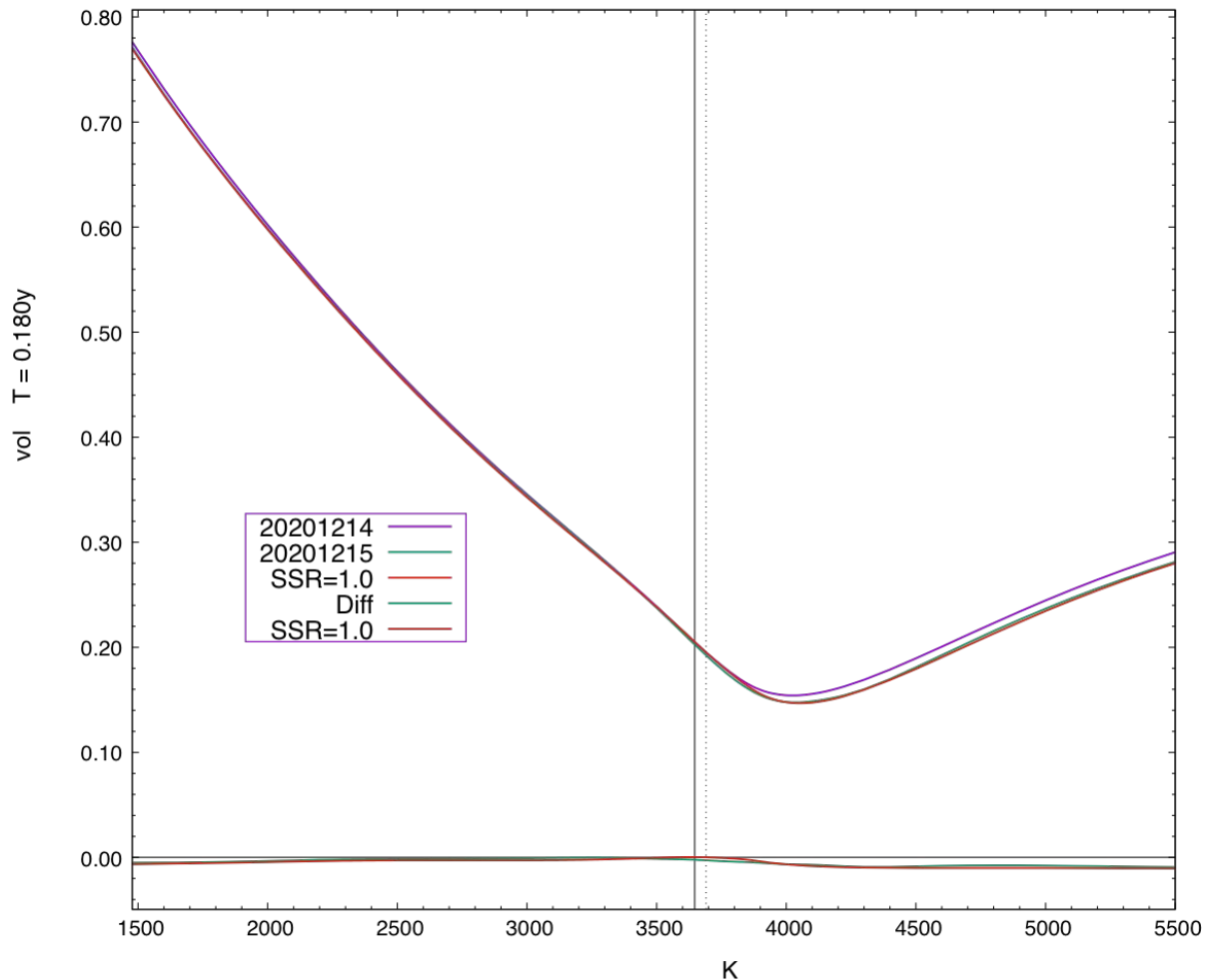
SSR=1, but NO sticky strike in the wings.

Instead: **Shapes are sticky-by-NS!**

Non-trivially so in the call and put wing!

This down-day comes after a sequence of (minor) down days, and SSR has mean-reverted to 1.0

SPX 20201214 to 20201215, return = 1.2%, T = 20210219

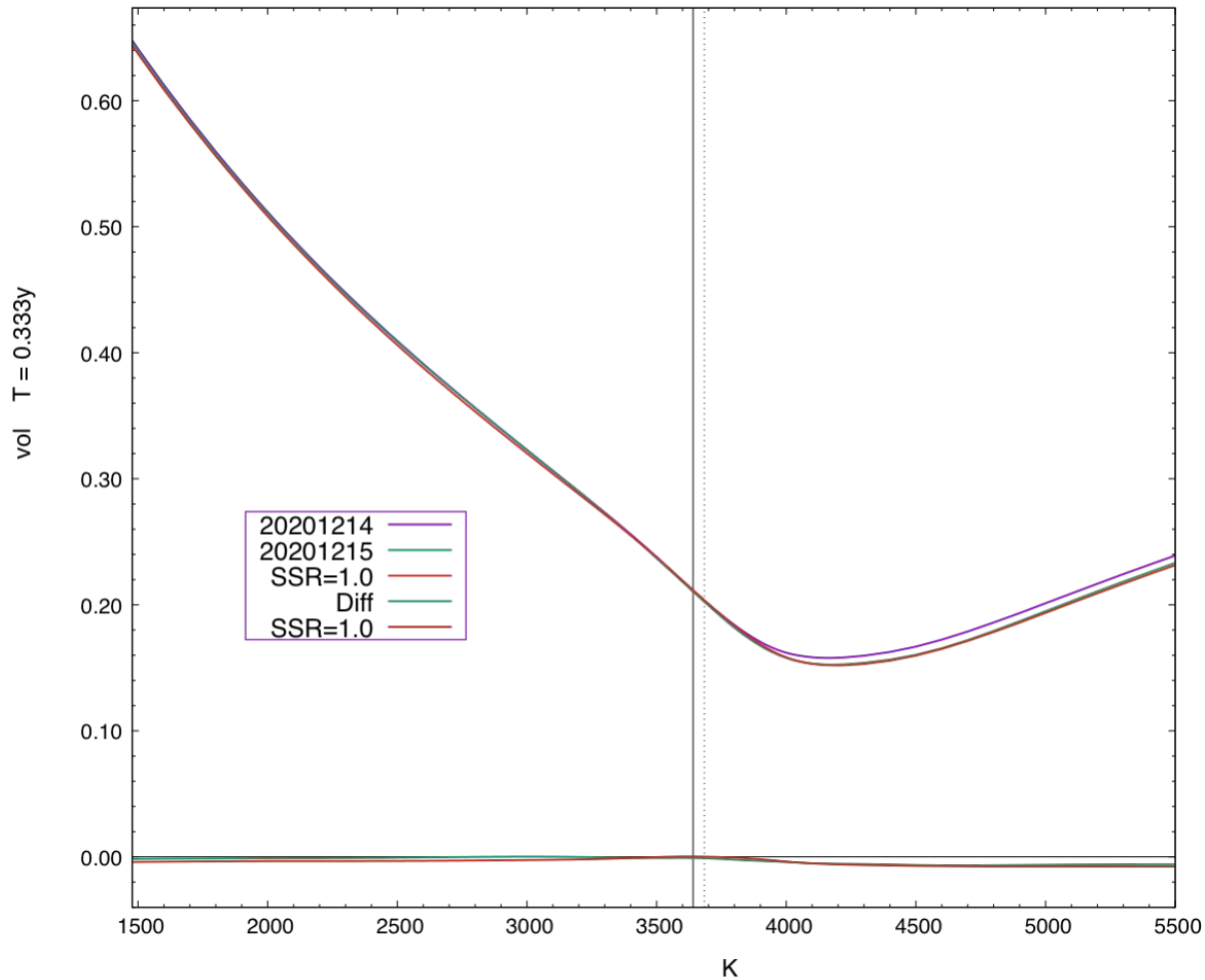


Close-to-close spot vol dynamics

SPX 2020-12-14 to 2020-12-15

T = 9w, SSR = 1

Fixed NS shape assumption
works **amazingly** well!

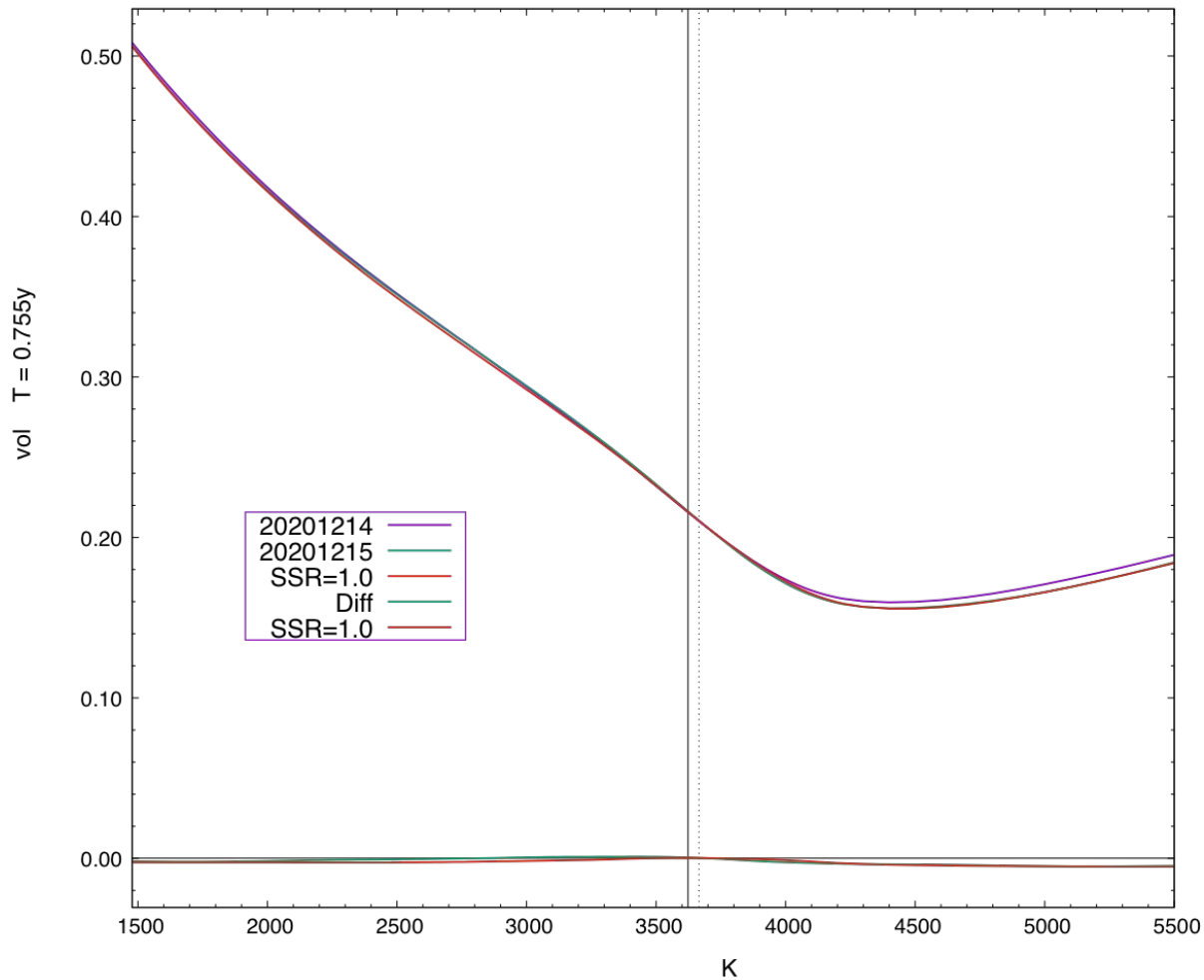


Close-to-close spot vol dynamics

SPX 2020-12-14 to 2020-12-15

$T = 4m$, $SSR = 1$

Fixed NS shape assumption
works **amazingly** well!

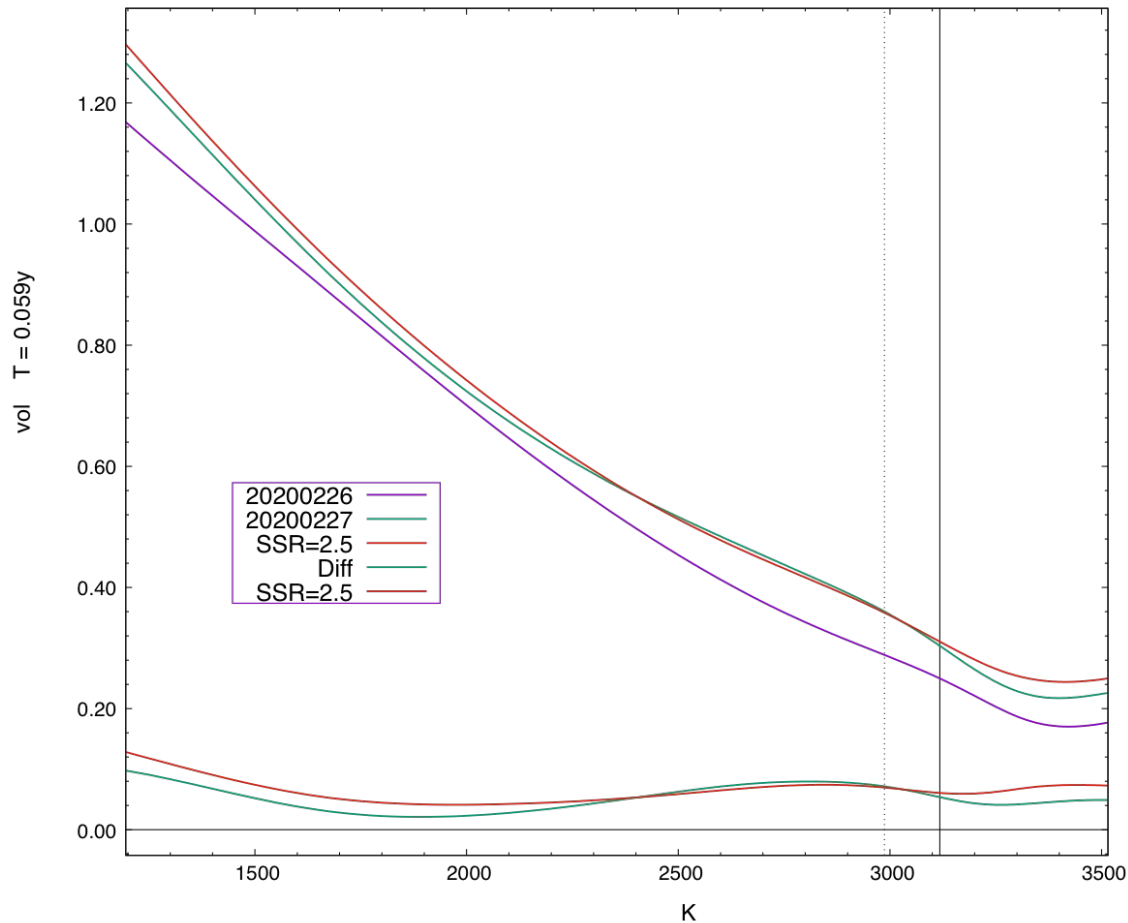


Close-to-close spot vol dynamics

SPX 2020-12-14 to 2020-12-15

$T = 9m$, $SSR = 1$

Fixed NS shape assumption
works **amazingly** well!



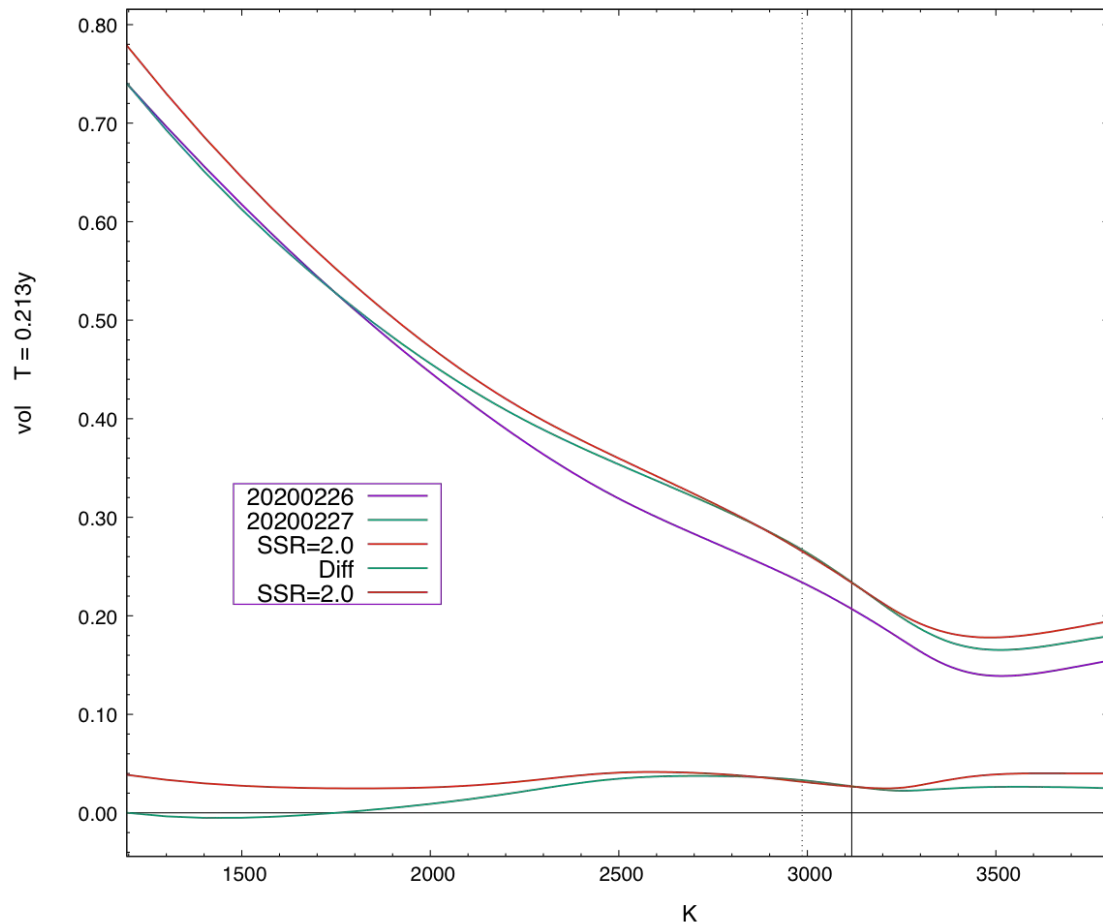
Close-to-close spot vol dynamics

SPX 2020-02-26 to 2020-02-27

Covid crash!

T = 3w, SSR = 2.5

Evidence for c2-spot-sensitivity > 0



Close-to-close spot vol dynamics

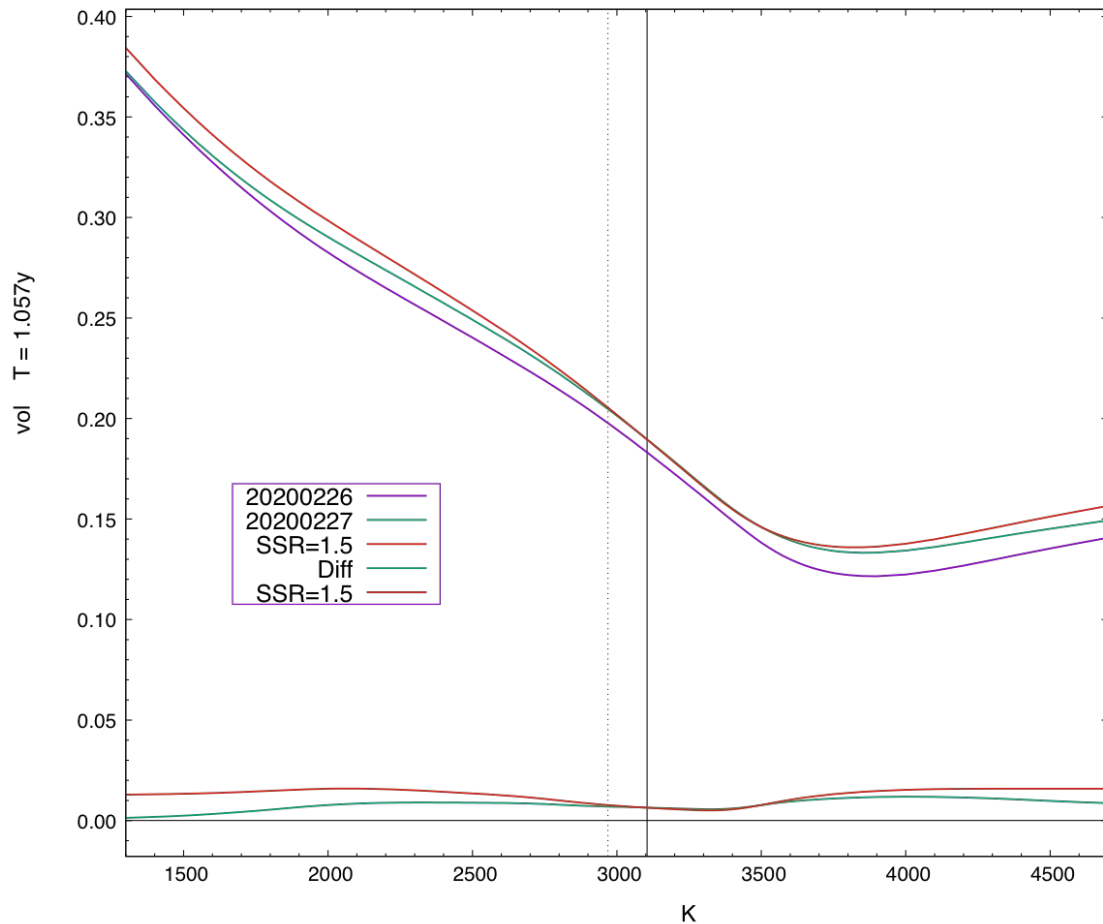
SPX 2020-02-26 to 2020-02-27

Covid crash!

$T = 2.5m$, $SSR = 2.0$

Evidence for $c2\text{-spot-sensitivity} > 0$

SPX 20200226 to 20200227, return = -4.2%, T = 20210319



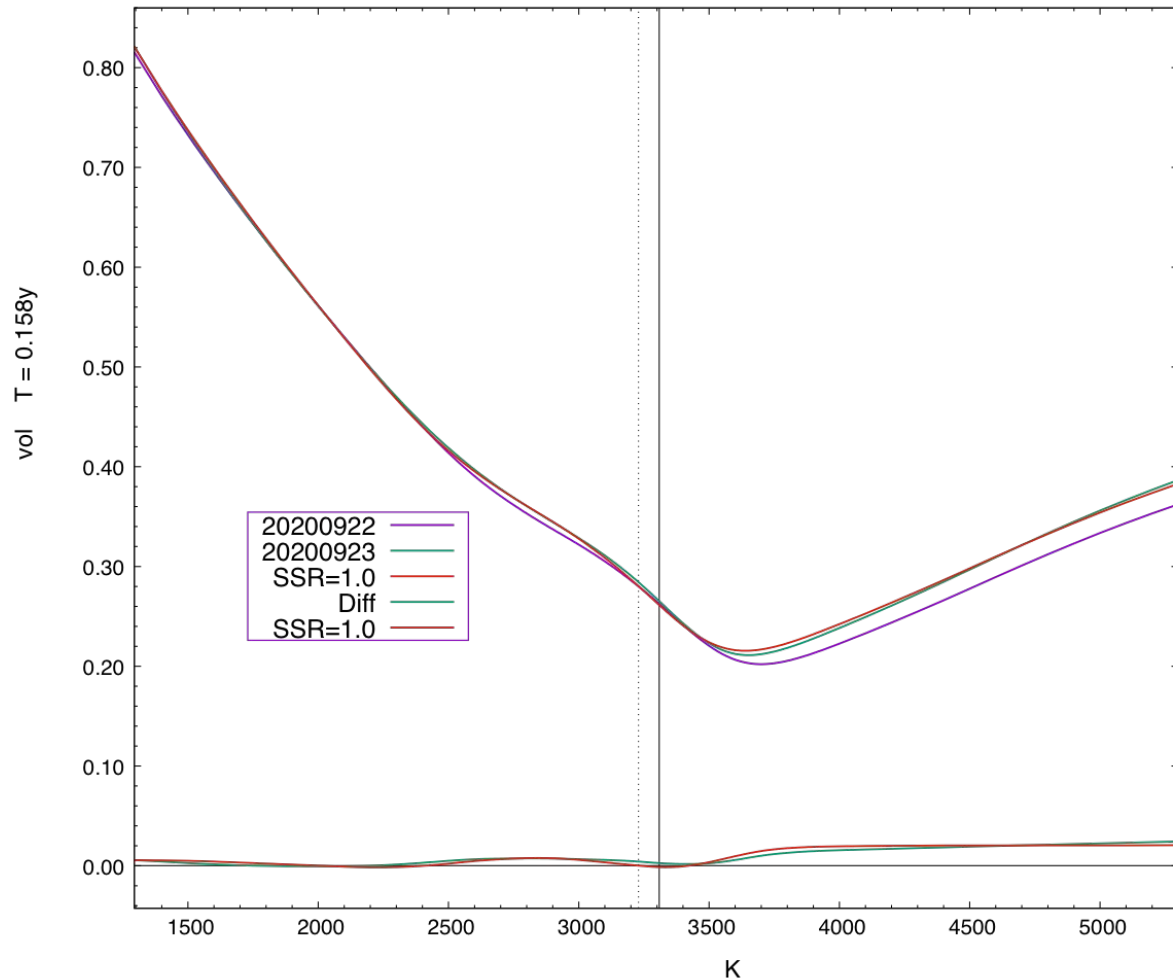
Close-to-close spot vol dynamics

SPX 2020-02-26 to 2020-02-27

Covid crash!

T = 1y, SSR = 1.5

Evidence for c2-spot-sensitivity > 0



Close-to-close spot vol dynamics

SPX 2020-09-22 to 2020-09-23

SSR=1, but NO sticky strike in the wings.

Instead: **Shapes are sticky-by-NS!**

Non-trivially so in the call and put wing!

This down-day comes after a sequence of (minor) down days, and SSR has mean-reverted to 1.0

Subtleties of Pricing American “vanillas”

- For European options (without divs) only integrated rates and variances matter.
 - Cash dividend modeling is relatively minor issue for Euro options (unless stochastic divs...).
- But **American** options are really path-dependent exotics and a lot of extra complications arise (esp. for ETFs, stocks, esp. with dividends):
- Empirically, in the olden days:
 - Could price every vanilla, European or American, with one flat r , q , and vol.
 - The same vol would work (well enough...) for call and put at same T, K .
- Already pretty hard, especially in real time. One needs:
 - A proper **cash dividend model** (no consensus even for vanilla...).
 - Handle **settlement** effects (incl. exchange and bank holidays).
 - A good choice of “**vol time**” (aka “trading/business time”), including “**events**”.
 - Then: imply “SPIBOR” (~daily), borrows (~real time), and vol surfaces (real time).
 - “**American PCP**” condition to imply borrow: Demand $\text{volP}(K) = \text{volC}(K)$ around ATM

Subtleties of Pricing American “vanillas” 2

- **Now:** How fancy does modeling have to be?
OR: What is the “**Market Model**” for American options (going to be...)?
 - BS: (1) Flat r, q, vol (2) $r(t), q(t), \text{vol}$ (3) $r(t), q(t), \text{vol}(t)$ for each $K(?)$
 - Beyond-BS: (4) $r(t), q(t), \text{LV}$, (5) $r(t), q(t), \text{SLV}$, (6) Other (approx/hacks...)
- Empirically in US: One definitely needs rate TS, vol-time incl. events, settlement, Berm (not Amer) exercise, proper dividend modeling, and “some” vol TS pricing.
- In Europe: Some evidence that LVs (or roughly equiv approx’s) are being used.
- Let’s look at some (more) examples:
 - Rate TS and event effects: TGT, MSFT, TSLA, CRWV
 - Settlement effects (+more): SPX
 - Settlement + vol TS: SPY

Event Time Effect on Pricing American Vanillas

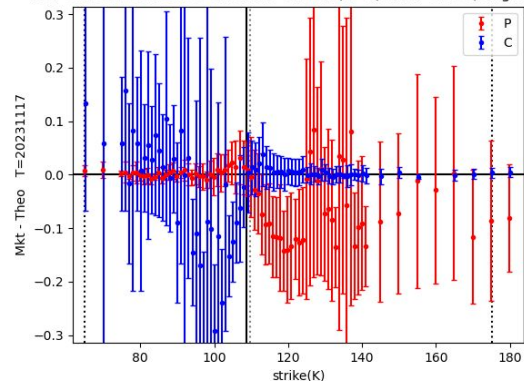
TGT 2023-11-08

Target has a dividend and earnings call just before expiry $T=2023-11-17$ ($i=1$).

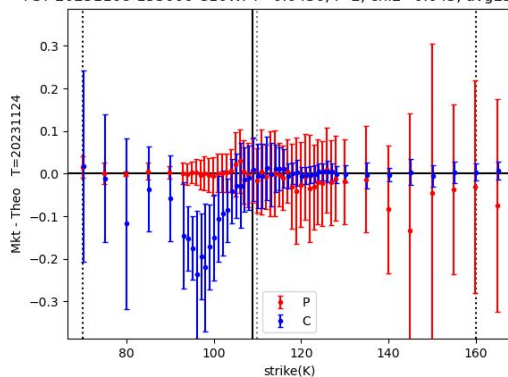
Top row: Without an “event time” an implied borrow allows (OTM and ITM) market prices to be matched at a few strikes, but not all.

Bottom row: With an **event time of 0.09y** all prices can be matched, in all expiries!

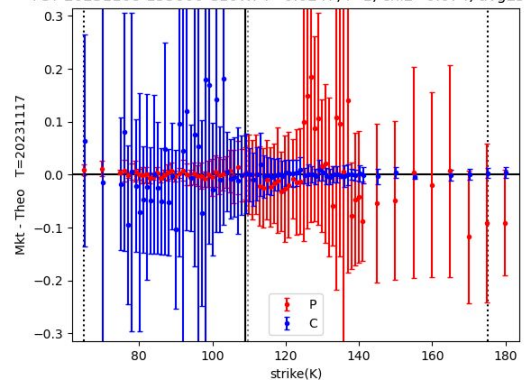
TGT 20231108-153000 C10W: $T=0.0247$, $i=1$, $\chi^2=0.086$, $\text{avgE5}=9.7$



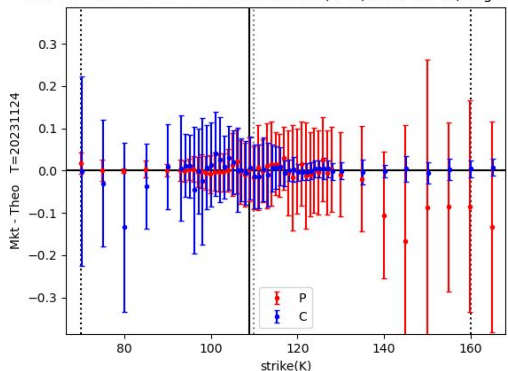
TGT 20231108-153000 C10W: $T=0.0436$, $i=2$, $\chi^2=0.045$, $\text{avgE5}=11.1$



TGT 20231108-153000 C10W: $T=0.0247$, $i=1$, $\chi^2=0.074$, $\text{avgE5}=2.3$



TGT 20231108-153000 C10W: $T=0.0436$, $i=2$, $\chi^2=0.043$, $\text{avgE5}=5.5$

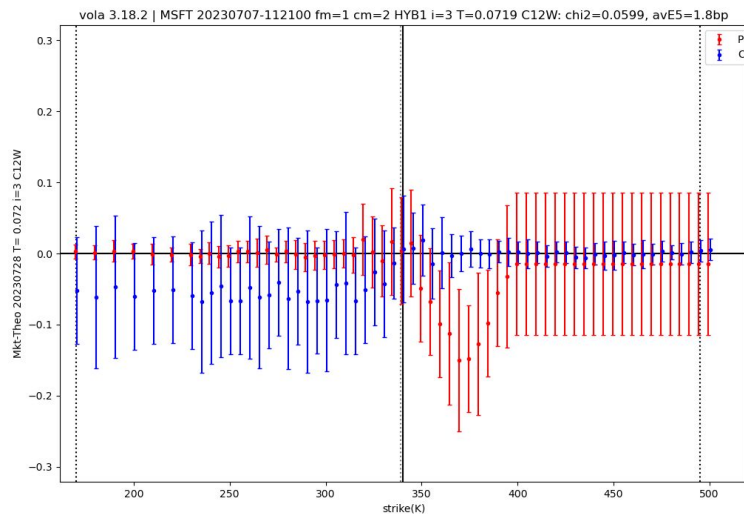


Rate TS and Event Time for American Vanillas

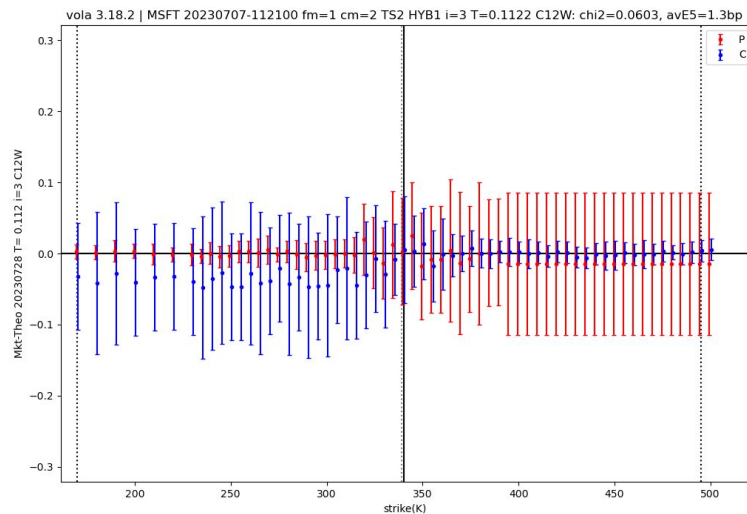
MSFT 2023-07-07

The ultimate test of a valuation approach is always the **price-difference plot**: Mkt - Theo

Flat term rates $r(T)$, $q(T)$



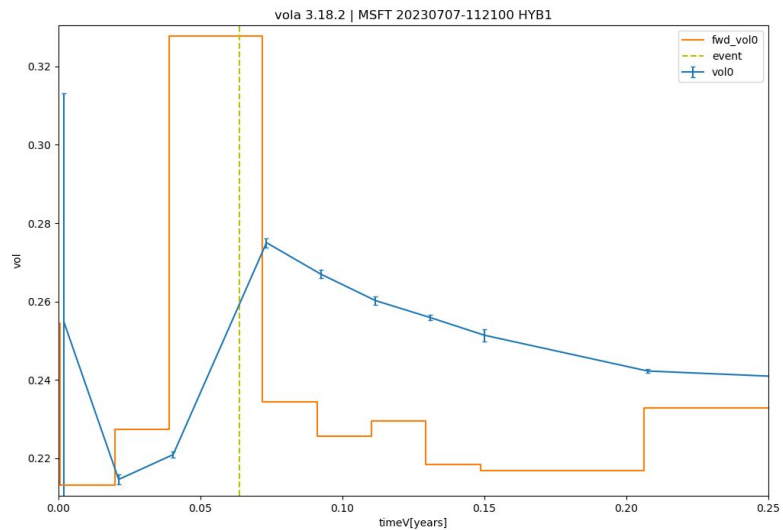
Local $r(t), q(t)$ and $\Delta T_E = 0.04y$



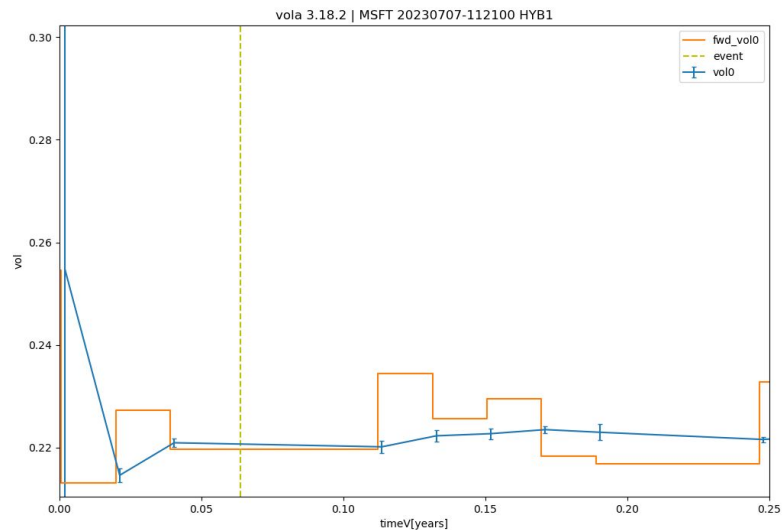
Rate TS and Event Time for American Vanillas

MSFT 2023-07-07

“Dirty” ATF vols

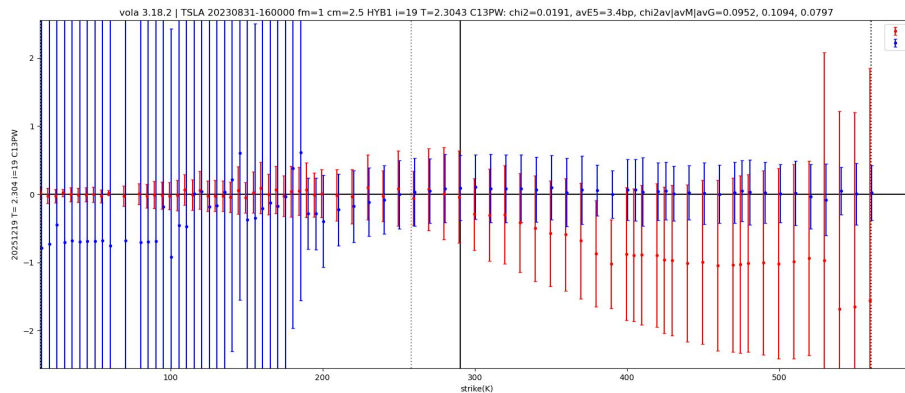


“Clean” ATF vols, $\Delta T_E = 0.04y$

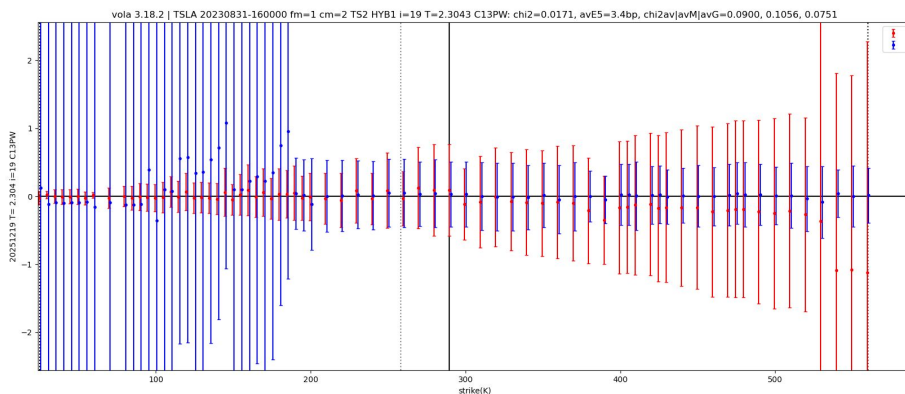


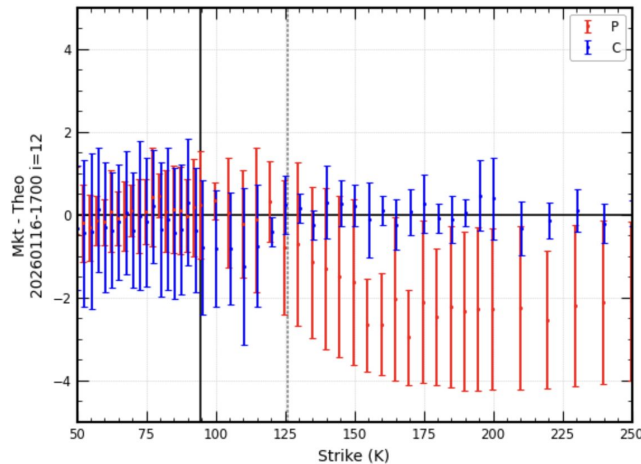
Rate Term-Structure Effect on Pricing American Vanillas

TSLA 2023-08-31



$T = 2.4y$

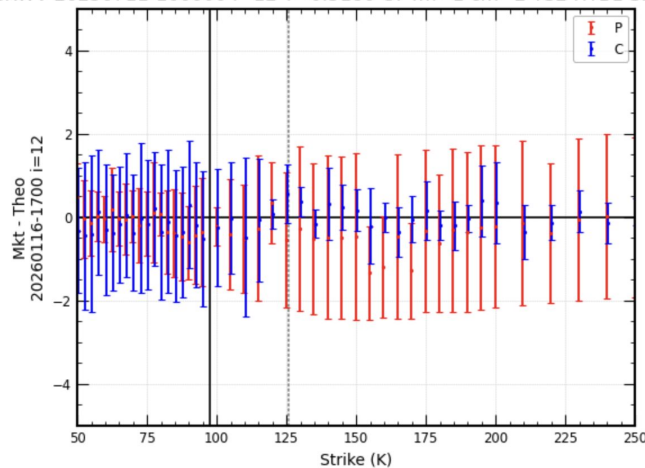




CRWV 2025-07-11

←TS0 priceDiff(K), T = 0.52y

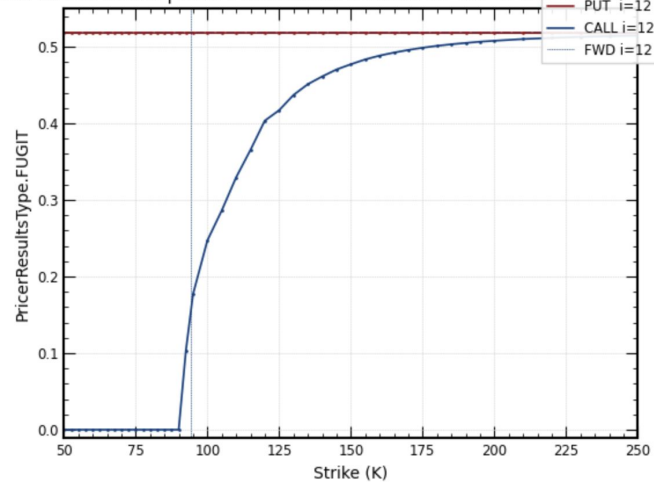
Flat rate pricing



←TS2 priceDiff(K), T = 0.52y

Local rate pricing

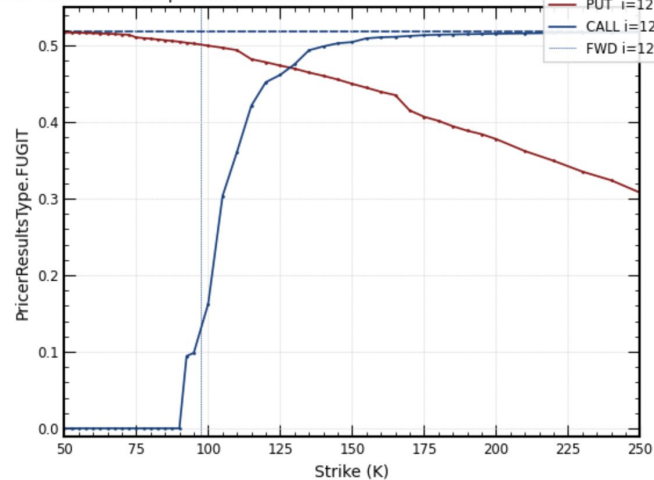
vola 3.22.0-beta0 | CRWV 20250711-160000 C7 fm=1 cm=2 TS0 HYB1 SM



CRWV 2025-07-11

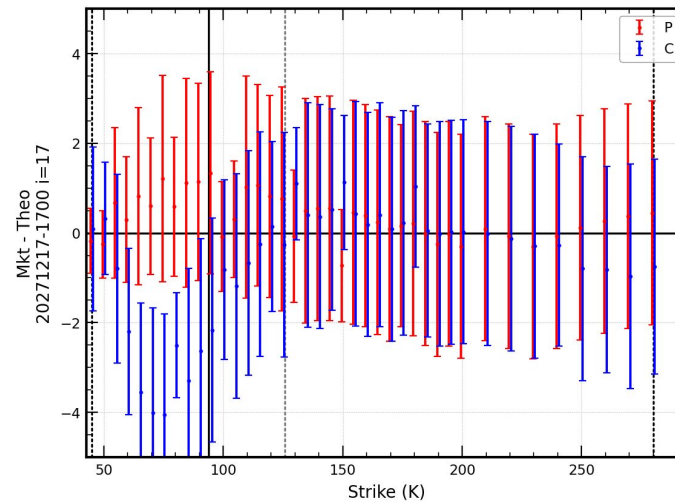
←TS0 fugit, $T = 0.52y$

vola 3.22.0-beta0 | CRWV 20250711-160000 C7 fm=1 cm=2 TS2 HYB1 SM



←TS2 fugit, $T = 0.52y$

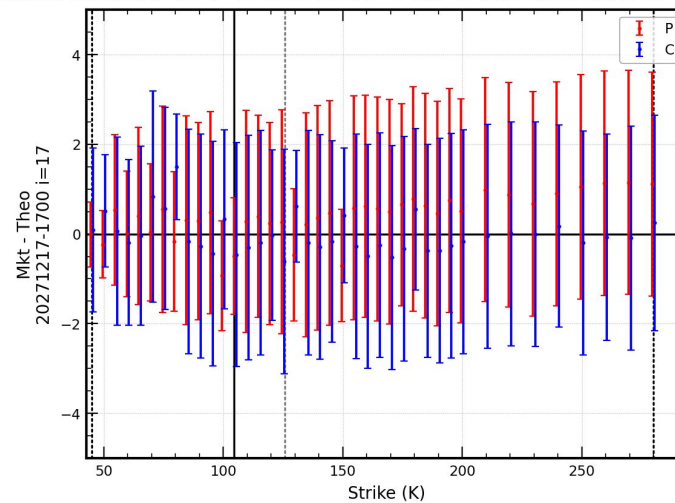
CRWV 20250711-160000 i=17 T=2.4326 C6 fm=1 cm=2 TS0 HYB1 SM chi2=0.0575 avgE5=43.3



CRWV 2025-07-11

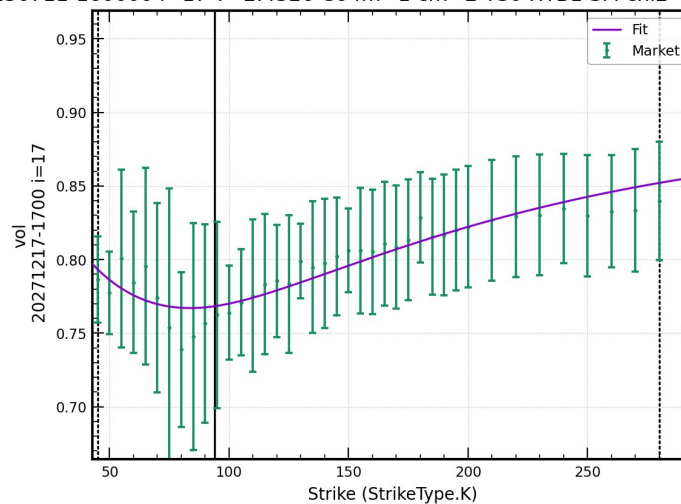
←TS0 priceDiff(K), T = 2.4y

CRWV 20250711-160000 i=17 T=2.4326 C6 fm=1 cm=2 TS2 HYB1 SM chi2=0.0217 avgE5=52.9



←TS2 priceDiff(K), T = 2.4y

CRWV 20250711-160000 i=17 T=2.4326 C6 fm=1 cm=2 TS0 HYB1 SM chi2=0.0575 avgE5=43.3

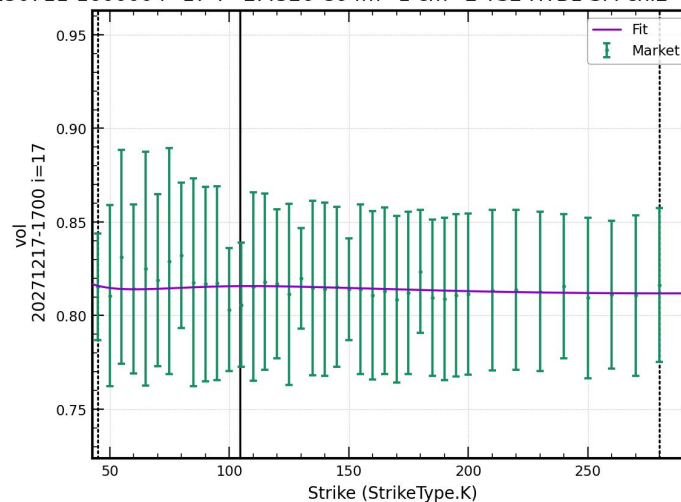


CRWV 2025-07-11

←TS0 fitted vol(K), T = 2.4y

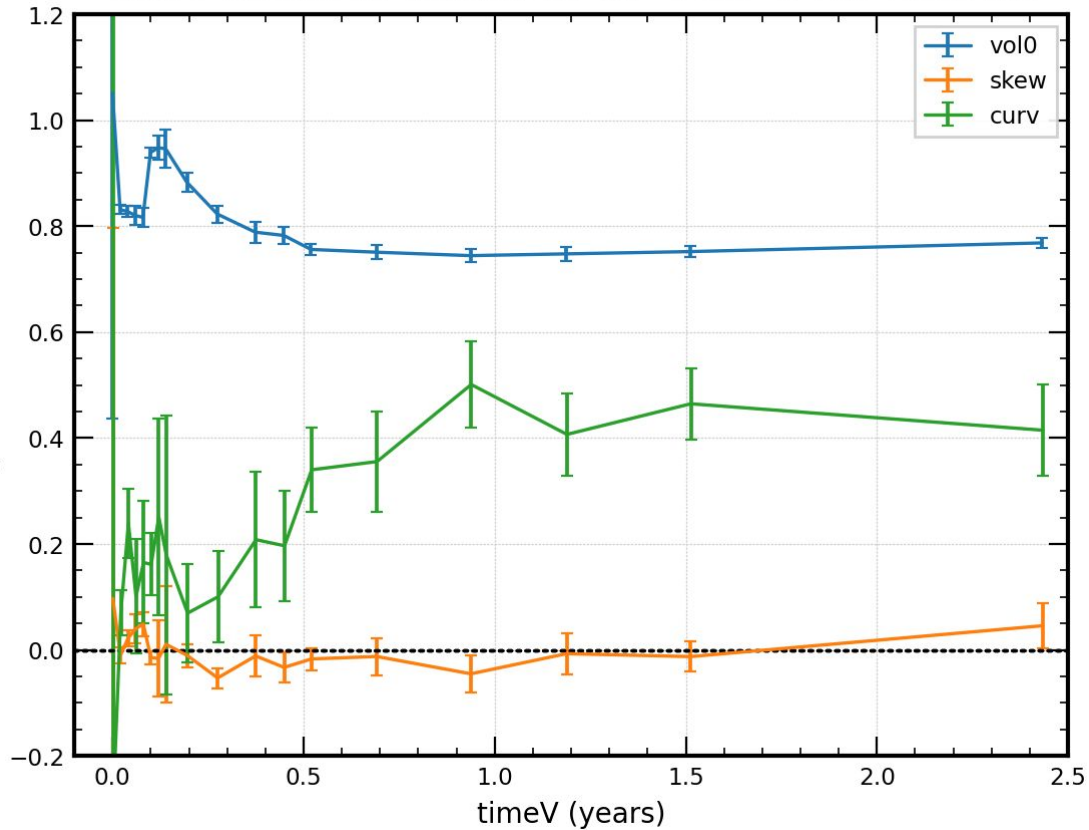
Note that implied forward is quite different with TS0 and TS2!

CRWV 20250711-160000 i=17 T=2.4326 C6 fm=1 cm=2 TS2 HYB1 SM chi2=0.0217 avgE5=52.9



←TS2 fitted vol(K), T = 2.4y

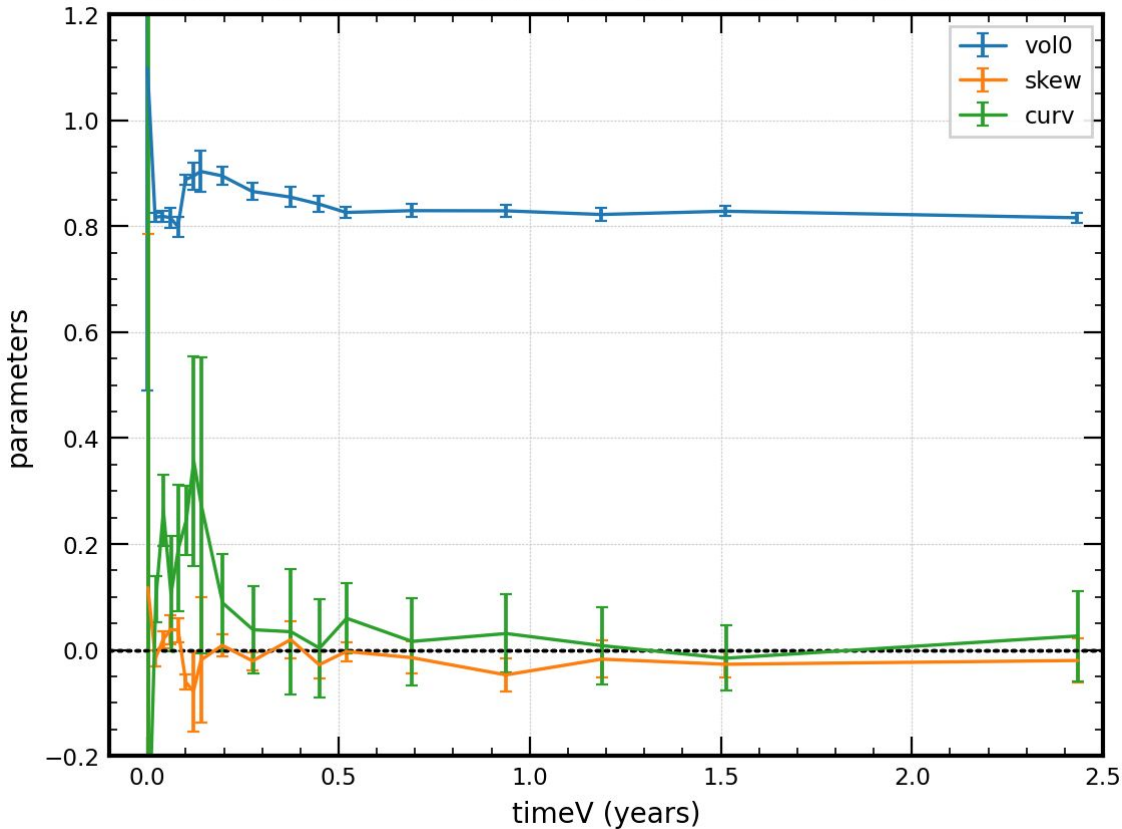
CRWV 20250711-160000 C6 fm=1 cm=2 TS0 HYB1 SM



CRWV 2025-07-11

Param TS (first 3) with **TS0**

CRWV 20250711-160000 C6 fm=1 cm=2 TS2 HYB1 SM

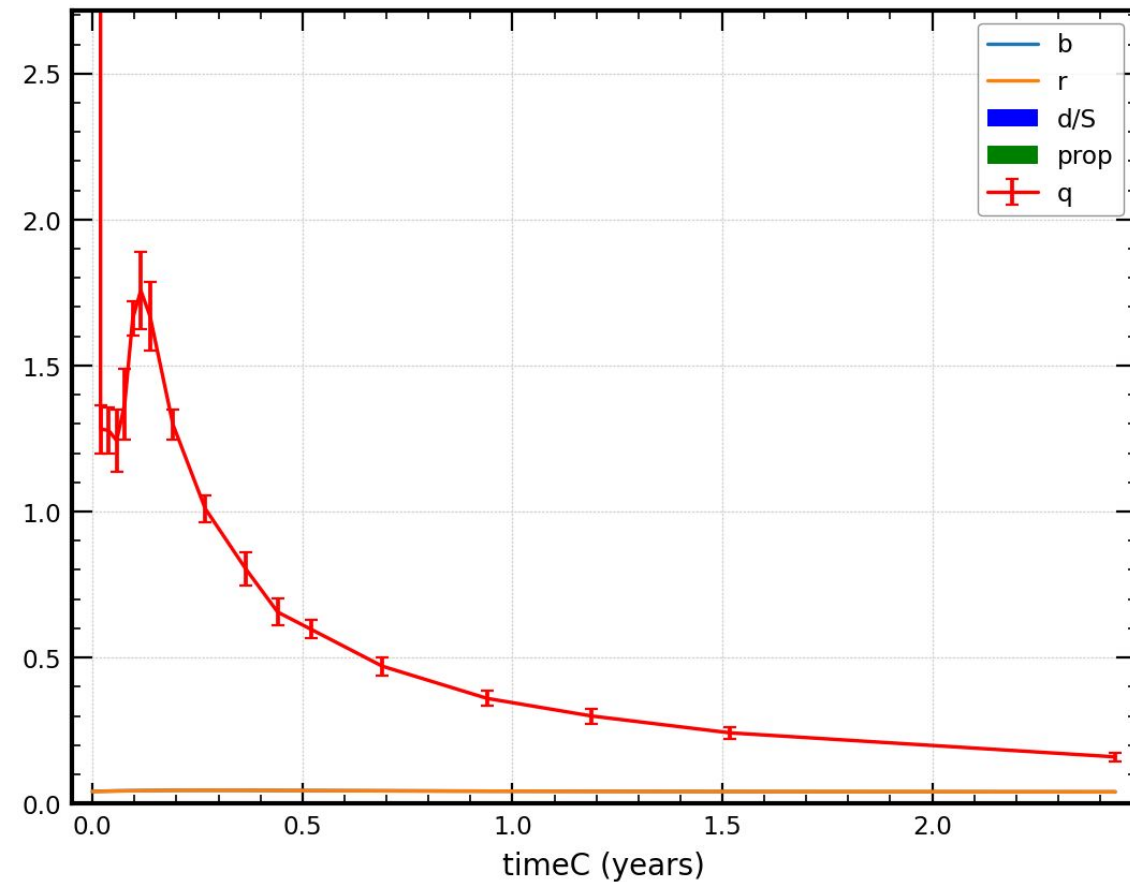


CRWV 2025-07-11

Param TS (first 3) with **TS2**

Very different from TS0 params,
eps. vol0 and c2.

CRWV 20250711-160000 C6 fm=1 cm=2 TS0 HYB1 SM

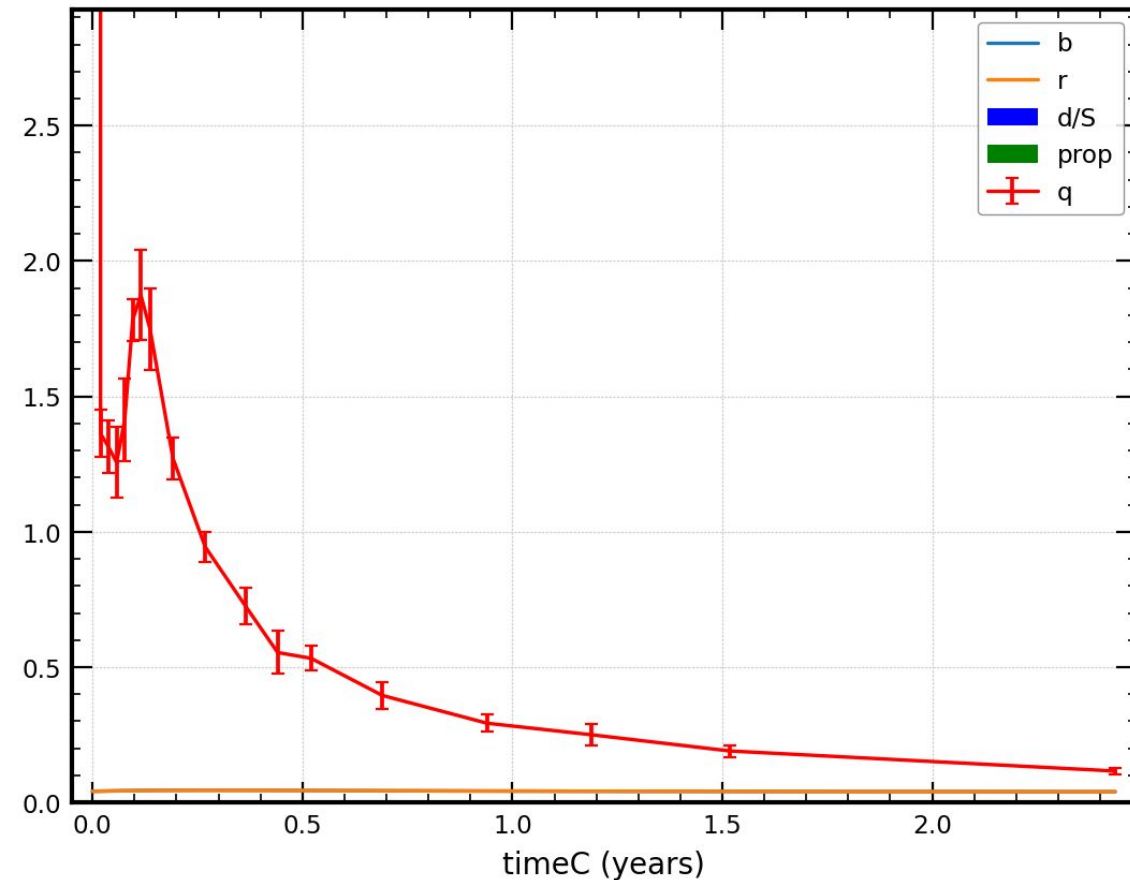


CRWV 2025-07-11

TS0 implied borrow $q(T)$

Non-monotonic !?

CRWV 20250711-160000 C6 fm=1 cm=2 TS2 HYB1 SM



CRWV 2025-07-11

TS2 implied borrow $q(T)$

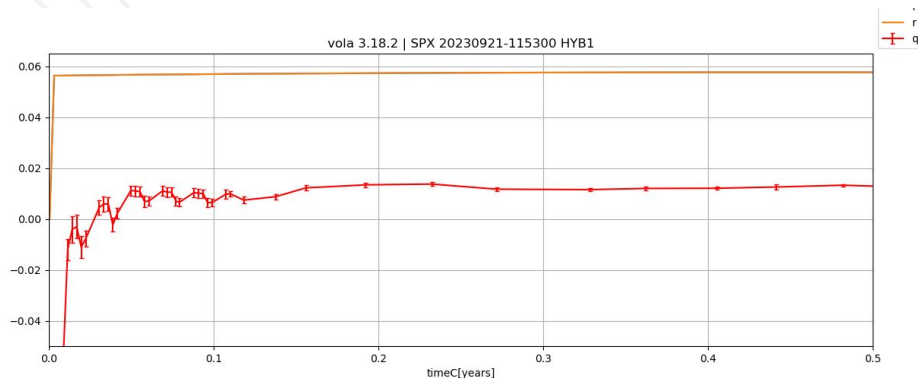
Also non-monotonic, but
otherwise similar to TS0 $q(T)$.

Lockup period ended 2025-08-14.

This $q(T)$ is really needed to match market!

Settlement Effects for SPX options

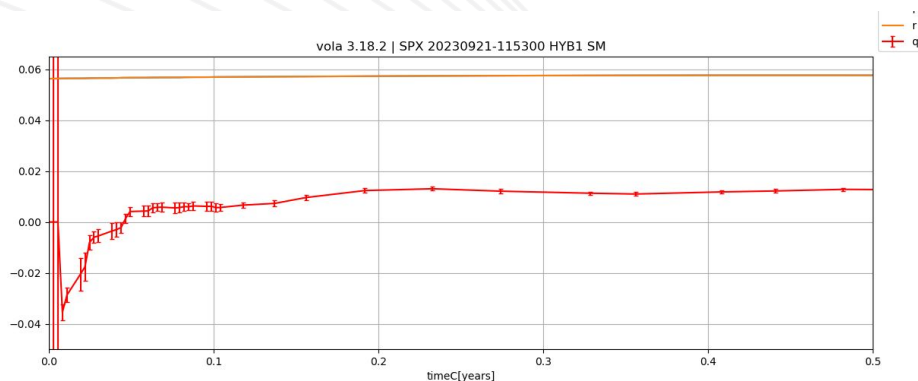
Let's treat SPX like an equity with a "spot", borrow cost, and (perhaps) cash dividends.



Implied borrow cost term structure

← Ignoring settlement, wrong spot

Wrong spot shows up as $1/T$ term in the borrow TS (made up wrong spot for illustration here...).

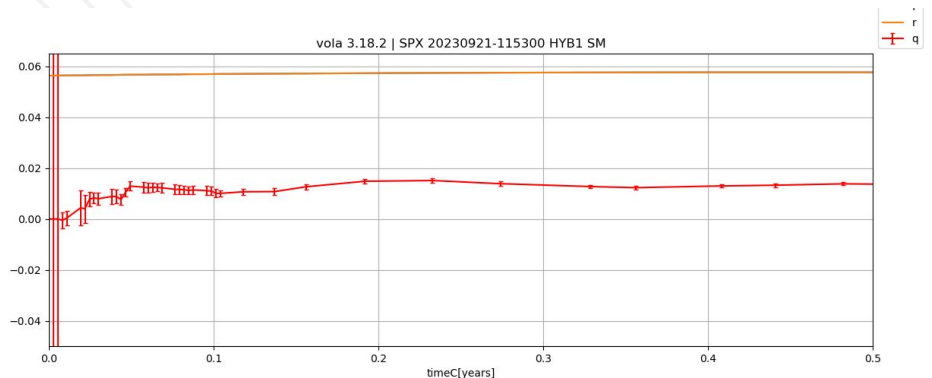


← With settlement, wrong spot

Now short-term borrow TS is smooth.

Settlement Effects for SPX options

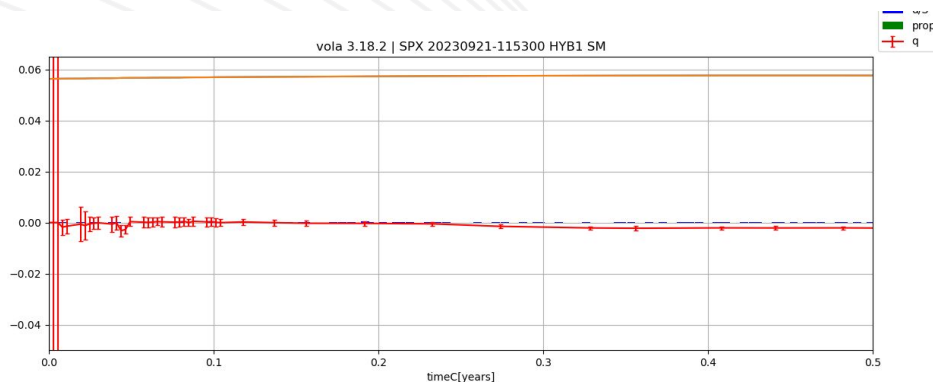
Let's treat SPX like an equity with a "spot", borrow cost, and (perhaps) cash dividends.



Implied borrow cost term structure

← With settlement, implied spot

No divs, so borrow includes div yield



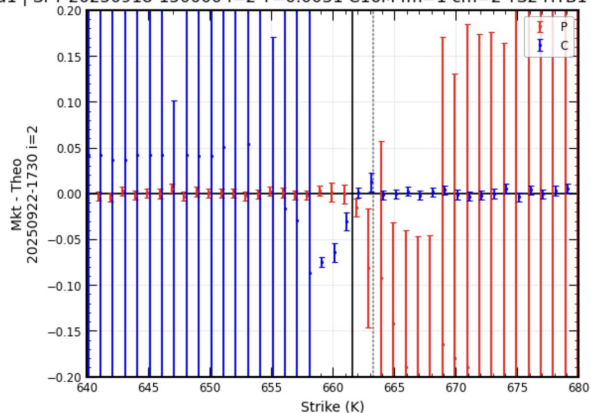
← With settlement, implied spot

With divs, so borrow is "pure" and very flat close to 0

Settlement + Vol Term-Structure Effects for **SPY** options

American options expiries right after dividends are hard:

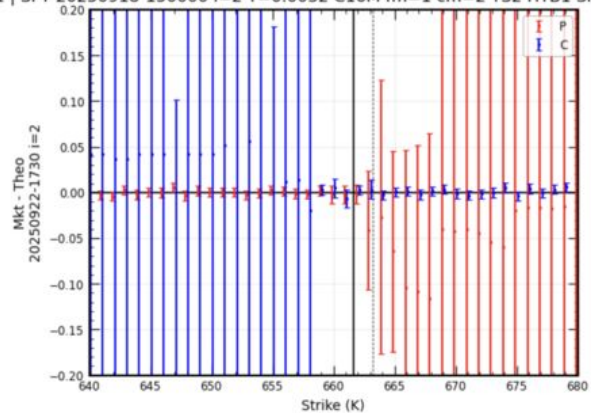
vola 3.22.0-beta1 | SPY 20250918-150000 i=2 T=0.0031 C16M fm=1 cm=2 TS2 HYB1 chi2=0.4023 avgE5=5.9



Price Difference plots:

← With **vol TS**, ignoring settlement

vola 3.22.0-beta1 | SPY 20250918-150000 i=2 T=0.0032 C16M fm=1 cm=2 TS2 HYB1 SM chi2=0.3938 avgE5=1.7



← With **vol TS**, and **settlement**



THE END

Questions?