

Tools for Options Trading in a Crazy World

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Timothy Klassen, PhD CEO/Founder, Vola Dynamics LLC

info@VolaDynamics.com



Vol Skews: 2008 versus 2020





Parameter TS: 2008 versus 2020





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Introduction and Summary



- Derivatives Trading has become (been forced to?) more transparent since 2008.
- The listed options market has grown dramatically over the last 20 years, incl. last!
- OTC flow and exotics markets can't ignore the listed (vanilla) market.
- Vanilla vol surfaces are the foundation of any derivatives business.
- Despite the liquidity and sophistication of the listed market it is not trivial to know at all times what vanillas are worth, or their greeks.
- It is a well-known "holy-grail" problem to produce sensible (arbitrage- and bias-free) theos/vol surfaces from the listed options market, especially in real-time (in any time, often...).
- We discuss the current state of the listed options market and illustrate why there are a lot of non-trivial data, modeling and algorithmic problems to solve.
- We will take an options/derivatives quant perspective.

Equity Options Markets Overview



- In US alone: circa 1,400,000 options on 4,600 underliers (just OPRA)
 - SPX has about 18,000 options (calls and puts) and about 40 expiries these days!
- Vanilla valuation is complicated due to dividends, borrow costs, events, and vol curves with lots of structure. Robustness is crucial, esp. in an HFT world.
- OMM: All options can only be valued with real-time, robust implied borrow curves and well-designed & calibrated volatility surfaces.
 - Also required for real-time risk, PnL decomposition/explain, margin, exotics, etc.
- All borrow and vol curve modeling and fitting analytics are proprietary.
 - In-house dev is a huge (ongoing) opportunity cost and business risk.
 - Now one vendor's library, Vola Dynamics, can help!

Implied Vols and Surfaces



- Implied volatility surfaces (and borrow cost curves) are the standard approach to summarizing the vanilla options market in an intuitive and compact manner.
- They provide the fundamental building block for the trading of vanillas (listed and OTC), as well as flow derivatives and exotics.
- There are many quant problems facing options and derivatives trading desks, but the long-standing problem of constructing sensible, arbitrage-free volatility surfaces from options market prices is perhaps the single hardest problem.

Implied Vols and Surfaces (cont'd)



- Before an implied vol can be calculated, other problems have to be solved:
 - Choice of "micro price" -- something better than inside bid and ask.
 - Handling of zero-bids.
 - Choice of "vol time" (aka "VTTX"), perhaps including "event time".
 - Dividend Modeling (no consensus even for Vanilla options!)
 - "De-Americanization" of American-exercise options (ETFs and stocks).
- See other talks, papers, and info on Vola Dynamics website (or ask...).
- NOTE: The problem in its purest form with less of these complications exists for (European exercise-style) SPX index options.

Vol Surface Parametrizations



- There are of advantages to having a vol curve parametrization per term with:
 - Intuitive parameters, as independent as possible, stable from fit to fit.
 - **Smooth** (in strike) over regions that are strongly correlated (cross-hedging...).
 - Little term-structure if possible (except on short end perhaps).
 - Makes it "easy" to avoid arbitrage, e.g. Lee bounds should be built in.
 - No hacks! (in wings, etc).
 - "SVJ" etc vol curves should be fittable within fraction of bp.
- A parametrization of the term-structure is not as crucial (it's also very hard...): one can T-interpolate and T-extrapolate good parametric curves quite easily.
 - But avoiding calendar arbitrage is crucial -- somehow curves have to be tied together.

Our parametrization approach

- Work one term at a time, impose smoothness across terms.
- Factor out overall vol level (ATF) as: $\sigma_0 := \sigma(T, K = F)$.
- Define "shape" curve $f(z) = f(z|\mathbf{p})$ as function of normalized strike $(NS)^1$

$$z := \frac{y}{\hat{\sigma}_0} = \frac{\log(K/F)}{\sigma_0\sqrt{T}}$$

such that

$$\sigma(z)^2 = \sigma_0^2 f(z|\mathbf{p})$$

 There are no standard definitions – we define dimensionless "skew" and "smile/convexity" as slope and curvature of shape curve:

$$f(z) =: 1 + \frac{s_2}{z} + \frac{1}{2} c_2 z^2 + \dots$$

• No butterfly arbitrage: Implied density ρ should be positive:

$$\hat{C}(T,K) = \int_0^\infty dS_T \ (S_T - K)_+ \ \rho_T(S_0 \to S_T)$$

$$\Rightarrow \quad \partial_{K}^{2} \hat{C}(T, K) = \rho_{T}(S_{0} \to S)|_{S=K}$$

- No calendar arbitrage: Total BS variance $w(y) := T\sigma(y)^2$ has to be increasing in T at any fixed y.
- Necessary (but generally not sufficient) constraint on the asymptotic wing behavior of implied vols (R. Lee, 2004):

$$w(y) \leq 2|y|$$
 as $|y| \to \infty$





- What are simplest possible implied vol curves? Need at least 3 parameters for ATF behavior.
- Vendors often use

$$\sigma(y)^n = \sigma_0^n \left(1 + s \ y + \frac{1}{2}c \ y^2\right) \quad \text{(or in terms of } z)$$

- Obviously has arbitrage in wings for n = 1, 2.
- Slight hope for n = 4, but would imply symmetric wings, which is intuitively and empirically wrong.
- Positivity has to be enforced too.
- Must do better...

Specific Curves: S3/SSVI

• Simplest sensible curve with 3 parameters $(c_2 \ge 0)$:

$$\sigma^{2}(z) = \sigma_{0}^{2} \left(\frac{1}{2}(1+s_{2}z) + \sqrt{\frac{1}{4}(1+s_{2}z)^{2} + \frac{1}{2}c_{2}z^{2}} \right)$$

- Was independently discovered by TRK (2003, "S3") and Gatheral/Jacquier (2013, "SSVI" = Simple SVI).
- Allows surprisingly varied skew shapes, including "takeover-for-cash" curves as c₂ → 0.
- Allows fitting of vast majority of US equity names.
- Very easy to avoid arbitrage (especially butterfly).
- In fact, in terms of the dimensionless variables $\hat{\sigma}_0, s_2, c_2$ can completely answer the butterfly-arbitrage question...



See our paper on SSRN for details about S3 curve, including simple necessary and sufficient no-butterfly arbitrage conditions in terms of parameters. Beyond the Simplest Curves: 5 Parameters (SVI, etc)



$$\sigma(z)^2 \rightarrow \sigma_0^2 C_{\pm} |z| \quad \text{as} \quad z \rightarrow \pm \infty \quad (\hat{\sigma}_0 C_{\pm} \le 2)$$

- For S3/SSVI: $C_{\pm} = \sqrt{\frac{1}{4}s_2^2 + \frac{1}{2}c_2} \pm \frac{1}{2}s_2$
- For Jim Gatheral's SVI and others (JW/L5, TRK) the C_{\pm} are independent parameters (constrained by $-C_{-} \leq s_2 \leq C_{+}$).
- Just some algebra to re-express their "raw" parametrization in terms of natural parameters σ₀, s₂, c₂, C₋, C₊. (Or minimum variance ratio instead of c₂.)
- Can fit some names better than with S3/SSVI.... but surprisingly not much better in many cases!?
- Certainly can not fit W-shaped curves around events (still $c_2 \ge 0$).





Beyond S3/SSVI, S5/SVI



- Liquid names can not be fit with simple curves like S3, S5, SABR (**S* curves**):
 - They have a unique maximum in their curvature around ATF and it is non-negative, which is not what the market always wants for liquid names.
 - Note e.g. that any kind of event (earnings, elections, Brexit, etc) leads to bi- or
 multi-nodal distributions, which can not be modeled by S* curves.
 - This is true not just for equity, but **also for FX, IR**.
- Need family of curves that allows more general curvature structures, including c2 < 0, but can be made arbitrage-free and fitted robust and fast.
- Vola Dynamics designed such a family of curves: **C* curves**: C6, C7, C8,, C16
- Let's look at some examples...



AAPL 20150721-154500 C8: T=0.0084, i=0, chi=0.248, avE5=13.1



AAPL 2015-07-21

C8 fit of W-shaped vol curves around earnings

Note "quotes in the middle of spread" in call wing...





VOLA DYNAMICS

AMZN 2018-04-26 earnings day

C8 Vol vs NS

Interesting Thursday: Earnings, new weekly listed, etc.

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VOLA DYNAMICS

Analytics for Options Trading

AMZN 2018-04-26 earnings day

Vol fit for first term, i=0, K-space

Most negative c2 ever!





AMZN 2018-04-26 earnings day

Vol fit for first term, i=0, NS-space

Most negative c2 ever!



VOLA DYNAMICS

AMZN 2018-04-26 earnings day

Vol fit for 2nd term, i=1, K-space

AMZN 20180426-154500 C8: T=0.0604, i=3, chi=0.035, avE5=2.2



= 20180518

-

Nol



AMZN 2018-04-26 earnings day

Vol fit for 4th term, i=3, K-space

Still negative c2!

AMZN 20180426-154500 C8: T=0.1371, i=7, chi=0.032, avE5=3.2





AMZN 2018-04-26 earnings day

Vol fit for 8th term, i=7, K-space

Flat around ATM now, c2≈0.

Use C10 if you worry about far wings...





AMZN 2018-04-26 earnings day

C8 parameter term-structure, first 3

Essentially flat shape params after 3m





Analytics for Options Trading

AMZN 2018-04-26 earnings day

C8 parameter term-structure

Essentially flat shape params after 3m





VOLA

Fitting **AEX** on day before Brexit (2016)

Vol vs NS

AEX on day before Brexit: T=2d, vols and implied density









Can non-W shapes be fitted with simple curves? No, not for liquid names!

SPX 20191104

SSVI / S3 fit, i=34, T=0.95y

This is a lousy fit even over a small range...

... even though shape looks "simple" (c2>0) and this is a supposedly easier longer term...

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SPX 20191104-153000 SABR3: T=0.9499, i=34, chi=121.976, avE5=16.3





SPX 20191104

SABR fit, i=34, T=0.95y

This is a lousy fit even over a small range...





T = 20201016

Vol



SPX 20191104

SVI / S5 fit, i=34, T = 0.95y

This is still a lousy fit even over a small range...

Ditto for T = 2y



Analytics for Options Trading

SPX 20191104

C15 fit, i=34, T = 0.95y

This is a great fit over a wide range, and can't be improved w/o over-fitting...

chi2 is 5000x smaller!

What is a "good" vol curve and fit ?

DYNAMICS

- The academic literature still has claims that e.g. SVI can fit SPX options.
- No... But how do we know what a "good" curve & fit is? Or a "good enough" one?
- Curves requirements:
 - Allow market vols to be fitted in arbitrage-free and **bias-free** manner.
 - Does this mean that fit has to go through all (inside) bid-ask spreads?
 - No, but has to be bias-free (on fraction of typical spread level) over time: requires very flexible curves
 - Usually 99.5% to 100.0% of theos should be within bid-ask spread for liquid names (⇔ bias-free).
- **Fitter** requirements are mostly technical but crucial in practice:
 - Has to find global minimum of some suitable objective function in a fast and robust fashion.
 - Curves, Fitter & Priors interact when it comes to producing arbitrage-free and sensible results w/o
 over-fitting -- the statistically best fit of the market is almost never the best answer...
 - Avoiding butterfly & calendar arbitrage in a fast and sensible manner require quite different algos.
- Simple curves like S* can definitely **not** fit liquid names in a **bias-free** manner.

Examples: Stable, Bias-Free, Arb-Free Fits



- Stability: Shape Curves tend to be extremely stable.
 - Intra- and extra-day unless "something really happens"...
- Good fits are "better than the market".
- To avoid calendar arbitrage, the fitter has to tie together different terms, separate term-by-term fits will never work...
- Let's look at some examples...





SPX 20190410 Shape Curve T = 0.7y

Example of fitter and shape **stability** -- even for **snapshot** fitting EOD over many days!

(No temporal filtering used here...)





SPX 20190417 Shape Curve T=1d

Example of fitter and shape **stability** -- **snapshot** fitting every 10 minutes for T=1d.

(No temporal filtering used here...)

No floppy wings!

AAPL 20191107-100000 C12m: T=0.1953, i=8, chi=1.176, avE5=2.0



T = 20200117

Vol



AAPL 20191107

Fit misses some strikes in CW and PW.

Should we trust market or fit? The fit looks biased.

Let's look later in day....

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AAPL 20191107-150000 C12m: T=0.1947, i=8, chi=0.121, avE5=2.0





AAPL 20191107

Looks like fit was right...





BYND 20200511

Do not trade off mids...






BYND 20200511

Do not trade off mids...

TSLA 20200403-150000 C12m: T=0.0386, i=2, chi=0.096, avE5=3.2





TSLA 20200403

Do not trade off mids...



T = 20201106

<u>ام</u>



TSLA 20201021

Different day -- very different shapes and spreads...





TSLA 20201021

Is the market using the Merton model ?

TSLA 20201021-150000 C12w: T=1.9043, i=16, chi=0.018, avE5=37.2



T = 20220916

No



TSLA 20201021

Is the market using the Merton model ?









Do not trade off mids...

NOTE: Strike range is 25x





AMZN 2018-04-26 earnings day

C8 total variance plot

First 10 terms

No calendar arbitrage! (Or butterfly...)

Interesting Thursday: Earnings, new weekly listed (i=6), etc.

Total Vars AMZN 20180426-154500 C8, chiAv=0.028, e5Av=6.7



≥



AMZN 2018-04-26 earnings day

C8 total variance plot

First 10 terms, with errors bars

Interesting Thursday: Earnings, new weekly listed (i=6), etc.







Fitting **AEX** on day before Brexit

Total Var plot with error bars

Curve Statistics for the OPRA Universe in 2020



- There are 4300 names in OPRA (Nov 2020). We find, roughly, for bias-free fits:
 - 3000 (70%) can be fit with S3 (aka SSVI).
 - 300 (7%) can be fit with S5 (aka SVI).
 - 600 (14%) can be fit with C6.
 - The remaining 400 (9%) names require higher C* curves.
- S5 is usually a temporary stop on the way from S3 to C6 or other C* curves.
- Among the top 100 names, perhaps 20% can be fit with S3 or S5. (None in top 25).
- SPX complex requires 14-16 parameters for some terms to get bias-free fits of all options down to zero-bids. Big tech names & global indices require 9-12 param's.
- Significantly more C* curves are needed in 2020, 2021 than earlier:
 - The Robinhood/Reddit crowd (Nasdaq Whale, etc?) have bid up the call wings, PMs are protecting gains via puts: Both wings are wide with structure (and close to fly arbitrage...).

Recent Funky Vol Curves Shapes

DYNAMICS

- W-shapes around earnings have existed since 2005 or so (GOOG)
- Since about 2010 even indices can have negative ATF curvature (c2 < 0).
 - Usually around events like FOMC, elections, Brexit, etc.
- In the Trump/Reddit era there is an additional never-ending stream of potential events, and the shape landscape has been taken to a new level.
- The shapes correspond to specific expectations about future underlier distributions -- the market has become quite sophisticated about these.
- Let's look at some recent examples (<u>LinkedIn posts</u> have more details on SPX in March 2020, and GME in Jan 2021).





SPX 2020-03-13 15:00

C15K, T = 1w, in NS-space

Very compressed call wing in NS.

If fit followed market in put wing more closely there would be fly arb...

SPX 20200313-150000 C15k: T=0.0186, i=3, chi=0.023, avE5=1.7



T = 20200320



SPX 2020-03-13 15:00

C15K, T = 1w, in K-space

If the fit followed market in put wing more closely there would be fly arb...







SPX 2020-03-13 15:00

C15K, T = 6w, in NS-space

Very compressed CW, very sharp knee...

SPX 20200313-150000 C15k: T=0.1152, i=18, chi=0.020, avE5=3.1



Vol T = 20200424



SPX 2020-03-13 15:00

C15K, T = 6w, in K-space



params

Parameter TS SPX 20200313-150000 C15k, chiAv=0.037, F0=2565.03



SPX 2020-03-13 15:00

C15K Param Term-Structure during the coronavirus crash

First 5 params

- Unprecedented c2 < 0 for ALL terms
- Super-steep near call wing: CW1

What do the funky shape curves mean?



- They correspond to specific expectations about the underlier evolution.
 - E.g. that SPX can't go up by more than 17% over one week, but can easily drop by 30%.
- These expectations are a lot more specific and sophisticated than e.g. during the GFC in 2008.

SPX 2020-03-13: Implied densities over 1w and 6w horizons





Parameter TS: 2008 versus 2020







Vol Skews: 2008 versus 2020





What's new with Spot-Vol Dynamics?

- How do vol surfaces move when the underlier moves?
 - Important for smart deltas, automatic theo updates, realistic scenarios, PnL explain, etc.
- "Sticky-Strike" or "Sticky-Delta" vol dynamics have not held for 15+ years.
 - Even on days when Sticky-Strike holds around ATM, it does not hold in wings!
- Mostly, **shapes** have been **sticky** by NS or Delta, but the ATF vol has moved according to a vol sensitivity, SSR, "super-skew", "vol sensi" etc on a steeper curve than the vol skew itself (ratio of slopes = SSR).
- SSR is between 1 and 2, usually, with a typical value of 1.3 to 1.5.
 - Roughly consistent with rough vol...
 - SSR has some horizon dependence from 1-min to 1-hour to 1-day (it's real...).
- New in 2020 (and for a few years before...)
 - SSR now often has clear term-structure. Moves around more too.
 - There is evidence for curvature (c2) sensitivity to spot, at least on daily horizons.

ATF Vol path (C8, volSensi = 1.5, clampFac = 0.2)

No



K, S



K, S

0.08

Vol sensi term-structure: SPX 20190805 1-min C12m chiTS(Pow)=0.183



pVol0



SPX 20190805

Vol sensitivity (SSR) term-structure

Parametric fit for robustness on small data sets (can be done intra-day)

Vol sensi term-structure: SPX 20200224 1-min C12m chiTS(Pow)=0.321





SPX 20200224

Vol sensitivity (SSR) term-structure

Parametric fit for robustness on small data sets







SPX 20210108

Vol sensitivity (SSR) term-structure

Parametric fit for robustness on small data sets







SPX 20200429

Vol sensitivity (SSR) term-structure

On up-days can be upward-sloping, and SSR < 1 at least for some terms SPX 20200226 to 20200227, return = -4.2%, T = 20200320





SPX 2020-02-26 to 2020-02-27

T = 3w, SSR = 2.5

Evidence for c2-spot-sensitivity > 0





SPX 2020-02-26 to 2020-02-27

T = 2.5m, SSR = 2.0

Evidence for c2-spot-sensitivity > 0



N



SPX 2020-02-26 to 2020-02-27

T = 1y, SSR = 1.5

Evidence for c2-spot-sensitivity > 0







SPX 20200227

Vol sensitivity (SSR) term-structure

On 1-min horizon



pVol0

Vol sensi TS: SPX 20200227 5-min C16m chiTS(Pow)=0.537, F[0]=3051.38



SPX 20200227

Vol sensitivity (SSR) term-structure

On 5-min horizon

Vol sensi TS: SPX 20200227 10-min C16m chiTS(Pow)=0.937, F[0]=3052.99





SPX 20200227

Vol sensitivity (SSR) term-structure

On 10-min horizon

One 1-day horizon even larger here, at least for short terms.

SPX 20200922 to 20200923, return = -2.4%, T = 20201120





SPX 2020-09-22 to 2020-09-23

Even when SSR = 1 does sticky-strike only hold around ATM, not in the wing(s):

Shapes are sticky-by-NS !!

This down-day comes after a sequence of (minor) down days, and SSR has mean-reverted/reversed to 1...

Spot-Vol Dynamics, Crazy Vol Shapes and Delta



- What is the **correct delta of a vanilla option**?
 - Delta (w/r/t F) = DeltaBlack + vega * dVol/dF
- dVdF (:= dVol/dF) and the delta adjustment are very large these days!
- dVdF can be calculated from the spot-vol dynamics.
 - Spot-Vol Dynamics is equivalent to knowing the optimal delta (hedges spot-correlated vol move).
- If shapes are stable just one dimensionless number (SSR) is needed.
- Fixed-strike dynamics, i.e. dVdF, and vol parameter dynamics (aka "vol path" for first parameter) behave qualitatively very differently (as we saw already)!
 - Only simple (robust) linear regressions are needed for parameter dynamics.
- For details, see our <u>LinkedIn post</u>.... Or briefly below...






SPX 20190805 T=0.13y M2

- Normalized dVol/dF
- Delta adjustments
- Final deltas

These dVdF (etc) curves are extremely stable across time, curve-type, algo details, etc.

Note: Fixed strike dVdF is plotted as a function of NS (using average F,T,vol0 over day).

Some firms use constant or linear approx for dVdF(K): Linear approx is fine in put wing, bad in call wing





SPX 20190805 T=0.13y M2

- Empirical regression of dvol vs dF for each strike, using 1-min data from 10:00 - 16:00
- Is consistent with SSR=1.5 and sticky-NS-shape dynamics over quite wide range!







SPX 20190805 T=0.13y M2

Super stable fit....

With steep "knee" at NS = +1.0

ATM parabola does not describe knee at all -- ATM curvature is negative!!

Explains break-down of linear approximation

SPX 20180122-150000 C15pm: T=0.0295, i=4, chi=0.061, pVol=1.0, pSkew=0.0





SPX 20180122 T=0.06y M1

- Normalized dVol/dF
- Delta adjustments
- Final deltas

Very different from 20190805, but still described well by just one number, SSR=1.0, and the precise vol curve fit...



F dVol/dF sqrt(T)

Empirical dVdF: SPX 20180122 1-min C12m, T=0.0297, i=4, pVol=0.97+-0.29



SPX 20180122 T=0.06y

- Empirical regression of dvol vs dF for each strike, using 1-min data from 10:00 - 16:00
- Is consistent with pVol=1.0 and sticky-NS-shape dynamics over quite wide range!
- And yes, dVdF can really be > 0 in put wing, even for indices.



Vol T = 20180202





SPX 20180122 T=0.06y

Super stable fit....

With steep "knee" at NS = +1.0

Here ATM parabola is not horrible in describing minimum, until NS=1.2 or so.

Questions arising for a bank desk when using sub-par curves

- Model Control/Valuation & Regulators would like the same surface/theos to be used across Flow, Exotics and OMM desks for a given name (one would hope...)
- How much time is spent massaging curves/surfaces?
 - A lot, it seems. Even then: A top tier bank had no SPX vol surface for 2 days in March...
- If the curves/surfaces are not flexible enough to match the market:
 - Actual "best" fit depends on weights put on different strike ranges. Not stable, will sometimes jump...
 - How to (bias-) correct? Different recipes for each product...
 - Even for var swaps: Is infinite-strip fair vol accurate? No. Is basis stable? Unlikely...

• Structured Products: Simple curves do not even match longer term market...

- How to hedge with vanillas? How to test that using simple curves for longer-dated SP does not lead to significant model error in valuation and risk? What happens once products are close to expiry?
- How important is proper spot-vol dynamics for exotics/SP deltas, vegas, etc?
- Can one trust a consensus pricing service for options valuation?



Consensus Pricing Service versus the listed AMZN market AMZN 2020-09-17, T = 1w





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Consensus Pricing Service versus the listed AMZN market AMZN 2020-09-17, T = 3m





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Questions?

Do you need tools for options and derivatives trading, in automated/electronic or any other fashion?

- Sophisticated prop shops, hedge funds, and banks rely on the Vola Dynamics quant library.
- See VolaDynamics.com, email info@VolaDynamics.com